NASA
Contractor Report 4644

Army Research Laboratory
Contractor Report ARL–CR–221

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GRANT NAG3–1469
JANUARY 1995

National Aeronautics and
Space Administration

U.S. ARMY
RESEARCH LABORATORY

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Prepared for
Propulsion Directorate
U.S. Army Aviation Systems Command
and
Lewis Research Center
under Grant NAG3–1469

National Aeronautics and
Space Administration

Office of Management

Scientific and Technical
Information Program

1995

GENERATION AND COMPUTERIZED SIMULATION OF MESHING AND CONTACT OF MODIFIED INVOLUTE HELICAL GEARS

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Abstract

The design and generation of modified involute helical gears that have a localized and stable bearing contact, and reduced noise and vibration characteristics are described. The localization of the bearing contact is achieved by the mismatch of the two generating surfaces that are used for generation of the pinion and the gear. The reduction of noise and vibration will be achieved by application of a parabolic function of transmission errors that is able to absorb the almost linear function of transmission errors caused by gear misalignment. The meshing and contact of misaligned gear drives can be analyzed by application of computer programs that have been developed. The computations confirmed the effectiveness of the proposed modification of the gear geometry. A numerical example that illustrates the developed theory is provided.

Nomenclature

a	Parabola parameter (fig. 2(a))
a_c	Modification coefficient of pinion rack-cutter surface (fig. 5)
b	Slope of linear function (fig. 1(b))
E_{pc}	The shortest distance between the pinion and rack-cutter Σ_c (fig. 6(b))
E_{pg}	The shortest distance between the pinion-gear axes (fig. 7)
E_{gt}	The shortest distance between the gear and rack-cutter Σ_t (fig. 6(a))
m_{21}	Gear ratio
$\mathbf{M}_{ij}$	Coordinate transformation matrix (from S_j to S_i)
$\mathbf{n}_r$	Unit normal vector to rack-cutter surface Σ_r $(r = c, t)$
$\mathbf{n}_f^{(i)}$	Unit normal vector to surface Σ_i represented in coordinate system S_f $(i = p, g)$
N_i	Number of teeth of the pinion $(i = 1, p)$ or the gear $(i = 2, g)$
$\mathbf{N}_r$	Normal vector to rack-cutter surface Σ_r $(r = c, t)$
p_n	Circular pitch in normal section (fig. 3)
P_n	Diametral pitch in normal section
r_i	Radius of the pitch circle of the pinion (or gear) $(i = p, g)$
$\mathbf{r}_i$,$\mathbf{r}_i^*$	Position vector of surface Σ_i
$\mathbf{r}_f^{(i)}$	Position vector of surface Σ_i represented in coordinate system S_f
s,s_r	Displacement of rack-cutter Σ_r $(r = c, t)$ (fig. 6)
S_i	Coordinate system i
u_i,θ_i	Surface parameters of Σ_i $(i = p, g)$
u_r,l_r	Surface parameters of Σ_r $(r = c, t)$
$\mathbf{v}^{(r)}$	Velocity of rack-cutter surface point $(r = c, t)$
$\mathbf{v}^{(ij)}$	Relative velocity of surface Σ_i point with respect to surface Σ_j point
α_o	Normal pressure angle (fig. 3)
β_o	Helix angle on the pinion (gear) pitch cylinder (figs. 3 and 4)
δ	Elastic approach of pinion and gear tooth surfaces

ΔE	Change of center distance
$\Delta\lambda_o$	Change of pinion lead angle on the pitch cylinder
$\Delta\mathbf{q}$	Displacement of contact point caused by misalignment
$\Delta\gamma_x$	Misalignment angle formed by crossed gear axes (fig. 8(a))
$\Delta\gamma_y$	Misalignment angle formed by intersected gear axes (fig. 8(b))
$\Delta\phi_2$,$\Delta\psi_2$	Transmission error (fig. 2)
$\Delta\boldsymbol{\phi}_2$	Vector of the angle of compensating turn of gear 2
λ_o	Lead angle on pinion pitch cylinder
Σ_i	Pinion $(i = p)$ and gear $(i = g)$ tooth surfaces
Σ_r	Rack-cutter surfaces $(r = c, t)$
ϕ_i,ψ_i	Rotation angle of gear i $(i = 1, 2, p, g)$ (figs. 2 and 7)
ψ_{gt}	Rotation angle of gear being in mesh with the rack-cutter Σ_t (fig. 6(a))
ψ_{pc}	Rotation angle of pinion being in mesh with the rack-cutter Σ_c (fig. 6(b))

1. Introduction

Conventional helical involute gears are designed for transformation of rotation between parallel axes. Theoretically, the gear tooth surfaces are in line tangency at every instant, along a straight line that is a tangent to the helix on the gear base cylinder. However, the line contact of gear tooth surfaces can be realized only for an ideal gear drive. In reality, the crossing of axes of rotation (instead of being parallel) and errors of lead angle result in the so-called edge contact, as a specific instantaneous point contact caused by curve-to-surface tangency. Here, the contacting curve is the edge of the tooth surface of one of the mating gears and the contacting surface is the tooth surface of the other one.

Trying to avoid the edge contact, the manufacturers of helical gears use various methods of crowning (deviation) of the theoretical gear tooth surfaces. However, the applied methods of crowning have not been complemented with the analysis of transmission errors caused by misalignment. Our investigation shows that improper crowning may avoid edge contact but cannot avoid the appearance of transmission errors of the shape shown in fig. 1. The function of such transmission errors is piecewise, almost linear, and has the frequency equal to the cycle of meshing of one pair of teeth. The above mentioned transmission errors cause high vibration and noise and therefore such transmission errors must be avoided. This can be achieved by application of computer numerically controlled (CNC) machines that have opened new perspectives for generation of gear tooth surfaces with improved topology.

The intent of this paper is to describe a modified topology of low-noise involute helical gears that satisfies the following requirements:

(1) The noise and vibration of helical gears are reduced substantially by application of a predesigned function of transmission errors of a parabolic type (fig. 2). Such a function can absorb (see below) an almost linear function of transmission errors shown in fig. 1.

(2) The bearing contact is localized. Theoretically, the tooth surfaces are in tangency at every instant at a point instead of a line. The contact of gear tooth surfaces at every instant

is spread over an elliptical area due to elastic deformation of gear teeth. The dimension of the instantaneous contact ellipse can be controlled by choosing proper design parameters.

(3) The proposed gear tooth surfaces can be generated by two rack-cutters designed for generation of the pinion and gear, respectively. A nonlinear transmission function in the process for gear generation must be provided and this can be accomplished by application of the CNC machine. A linear transmission function is provided in the process for the pinion generation.

2. Interaction of Parabolic and Linear Function of Transmission Errors

The ideal gears transform rotation with constant gear ratio $m_{21} = \frac{N_1}{N_2}$, and the ideal transmission function is

$$\phi_2^o(\phi_1) = \frac{N_1}{N_2}\phi_1 \tag{1}$$

where N_1 and N_2 are the tooth numbers of the pinion and gear, respectively.

However, the crossing of gear axes (instead of being parallel), intersection of these axes and errors of lead angle cause a transmission function $\phi_2(\phi_1)$ that is shown in fig. 1(a). Our investigation (see sections 4-6) shows that the function of transmission errors caused by above mentioned errors of misalignment is a piecewise almost linear function of transmission errors $\Delta\phi_2(\phi_1)$ with the frequency of a cycle of meshing for one pair of teeth (fig. 1(b)).

Here:

$$\Delta\phi_2(\phi_1) = \phi_2(\phi_1) - \frac{N_1}{N_2}\phi_1 \tag{2}$$

Transmission errors of this type cause a discontinuity of the transmission function and a

big jump of the angular velocity of the driven gear at transfer points (when one pair of teeth is changed to another one). Therefore, vibration and noise become inevitable.

It was proven [1,2,4] that a predesigned parabolic function of transmission errors interacting with a linear function will become a parabolic function with the same parabola parameter. A parabolic function of transmission errors is much more preferable than a linear function since the transmission function will be a continuous one, the jump of angular velocity of the driven gear and the stroke at the transfer point will be substantially reduced.

Fig. 2(a) shows the sum of two functions of transmission errors

$$\Delta\phi_2(\phi_1) = \Delta\phi_2^{(1)}(\phi_1) + \Delta\phi_2^{(2)}(\phi_1) = b\phi_1 - a\phi_1^2 \tag{3}$$

The first one, $\Delta\phi_2^{(1)}(\phi_1)$, is caused by misalignment. The second one, $\Delta\phi_2^{(2)}(\phi_1)$, is a predesigned parabolic function which exists even if misalignment does not appear. It is easy to verify that equation (3) represents in the new coordinate system $(\Delta\psi_2,\ \psi_1)$ the parabolic function (fig. 2(b)) that is designated as

$$\Delta\psi_2 = -a\psi_1^2 \tag{4}$$

The parabola parameter a in equations (3) and (4) is the same. Axes of coordinate system $(\Delta\psi_2,\ \psi_1)$ and $(\Delta\phi_2,\ \phi_1)$ are parallel but the origins are different. The coordinate transformation from $(\Delta\phi_2,\ \phi_1)$ to $(\Delta\psi_2,\ \psi_1)$ is represented with the following equations

$$\Delta\psi_2 = \Delta\phi_2 - \frac{b^2}{4a}, \quad \psi_1 = \phi_1 - \frac{b}{2a} \tag{5}$$

The difference between functions $\Delta\phi_2(\phi_1)$ and $\Delta\psi_2(\psi_1)$ is the location of the couple of points (A, B) and the respective points (A^*, B^*) (fig. 2(a)). The symmetrical location of (A, B) is turned into the asymmetrical location of (A^*, B^*). However, the interaction of several functions $\Delta\psi_2(\psi_1)$ determined for several tooth surfaces being in mesh may provide a symmetrical parabolic function of transmission errors as shown in fig. 2(b). This can be achieved if the parabolic function $\Delta\phi_2(\phi_1)$ will be predesigned in the area (fig. 2(a))

$$\phi_1(B) - \phi_1(A) \geq \frac{2\pi}{N_1} + 2c \tag{6}$$

where $c = \dfrac{b}{2a}$. Requirement (6), if observed, enables to provide a continuous function $\Delta\psi_2(\psi_1)$ for the range of $\dfrac{2\pi}{N_1}$ where N_1 is the pinion tooth number. It will be shown below (see sections 5 and 6) that functions of transmission errors caused by angular errors (such as the crossing and intersection of the axes of rotation, error of the lead angles) are indeed piecewise linear functions, and the coefficient b can be determined knowing the angular error caused by misalignment and the design parameters of the gear drive.

3. Surfaces of Rack-Cutters

The imaginary process of generation of conjugate tooth surfaces is based on application of two rack-cutters that are provided respectively by a plane Σ_t and a cylindrical surface Σ_c that differs slightly from plane Σ_t (see fig. 3). The rack-cutter surfaces Σ_t and Σ_c are rigidly connected each to other in the process of the imaginary generation, and they are in tangency along a straight line, $O_b z_b$ (fig. 5). This line and the parallel axes of the gears form angle β_o, that is equal to the helix angle on the pinion (gear) pitch cylinder. The normal sections of the rack-cutters are shown in figs. 3 and 5. Rack-cutter surface Σ_c generates the pinion tooth surface Σ_p, and the rack-cutter surface Σ_t generates the gear tooth surface Σ_g.

Gear Rack-Cutters Σ_t

Using figs. 3, 4 and 5, we represent the transformation matrix from system S_a to S_r $(r = c, t)$ and S_b to S_a as follows

$$\boldsymbol{M}_{ra} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta_o & \sin\beta_o & 0 \\ 0 & -\sin\beta_o & \cos\beta_o & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$$\boldsymbol{M}_{ab} = \begin{bmatrix} \cos\alpha_o & -\sin\alpha_o & 0 & -d_p\cos\alpha_o \\ \sin\alpha_o & \cos\alpha_o & 0 & a_m - d_p\sin\alpha_o \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

$$\boldsymbol{M}_{rb} = \begin{bmatrix} \cos\alpha_o & -\sin\alpha_o & 0 & -d_p\cos\alpha_o \\ \sin\alpha_o\cos\beta_o & \cos\alpha_o\cos\beta_o & \sin\beta_o & (a_m - d_p\sin\alpha_o)\cos\beta_o \\ -\sin\alpha_o\sin\beta_o & -\cos\alpha_o\sin\beta_o & \cos\beta_o & -(a_m - d_p\sin\alpha_o)\sin\beta_o \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

Here α_o and β_o are the normal pressure angle and the helix angle of the rack-cutter; a_m is the half of the tooth width of the rack-cutter on middle line $m - m$ (fig. 3), where

$$a_m = \frac{\pi}{4P_n} \tag{10}$$

and P_n is the normal diametral pitch of the rack-cutter, d_p is the distance between middle line $m - m$ of the rack-cutter and the origin O_b along axis x_b, fig. 5. Parameter d_p can be controlled to adjust the location of the contact path on the gear tooth surface.

Surface Σ_t of the gear rack-cutter is a plane that is represented in S_b as

$$\mathbf{r}_t^{(b)} = \begin{bmatrix} u_t & 0 & l_t \end{bmatrix}^T \tag{11}$$

where $(u_t,\ l_t)$ are the surface parameters.

Rack-cutter surface Σ_t is represented in coordinate system S_t by the matrix equation

$$\mathbf{r}_t(u_t, l_t) = \mathbf{M}_{tb}\mathbf{r}_t^{(b)} \tag{12}$$

Equations (9), (11) and (12) yield

$$\mathbf{r}_t = \begin{bmatrix} (u_t - d_p)\cos\alpha_o \\ [(u_t - d_p)\sin\alpha_o + a_m]\cos\beta_o + l_t\sin\beta_o \\ -[(u_t - d_p)\sin\alpha_o + a_m]\sin\beta_o + l_t\cos\beta_o \end{bmatrix} \tag{13}$$

The unit normal to Σ_t is represented in S_t by equations

$$\mathbf{n}_t = \frac{\mathbf{N}_t}{|\mathbf{N}_t|}\ , \quad \mathbf{N}_t = \frac{\partial \mathbf{r}_t}{\partial l_t} \times \frac{\partial \mathbf{r}_t}{\partial u_t} \tag{14}$$

that yield

$$\mathbf{n}_t = \begin{bmatrix} -\sin\alpha_o & \cos\alpha_o\cos\beta_o & -\cos\alpha_o\sin\beta_o \end{bmatrix}^T \tag{15}$$

Pinion Rack-Cutter Surface Σ_c

Rack-cutter Σ_c generates the pinion. The normal section of rack-cutter surface Σ_c (fig. 5) is a parabolic curve. We remind that the normal section of rack-cutter surface Σ_t is a straight line directed along axis x_b in fig. 5. The parabolic curve is in tangency with the x_b-axis at point $N(O_b)$. Rack-cutter surfaces Σ_c and Σ_t are in tangency along a straight line that is parallel to axes z_a and z_b and passes through point O_b that coincides with point N. The deviation of the parabolic curve from the x_b-axis affects the dimensions of the instantaneous contact ellipse.

Rack-cutter surface Σ_c is represented in S_b as follows

$$\mathbf{r}_c^{(b)} = \begin{bmatrix} u_c & -a_c u_c^2 & l_c \end{bmatrix}^T \tag{16}$$

where a_c is coefficient of the parabolic normal section, and (u_c, l_c) are the surface parameters of Σ_c.

Rack-cutter surface Σ_c is represented in coordinate system S_c by the matrix equation

$$\mathbf{r}_c(u_c, l_c) = \mathbf{M}_{cb}\mathbf{r}_c^{(b)} \tag{17}$$

Equations (9), (16) and (17) yield

$$
\mathbf{r}_c = \begin{bmatrix} (u_c - d_p)\cos\alpha_o + a_c u_c^2 \sin\alpha_o \\ \\ [(u_c - d_p)\sin\alpha_o + a_m]\cos\beta_o - a_c u_c^2 \cos\alpha_o \cos\beta_o + l_c \sin\beta_o \\ \\ -[(u_c - d_p)\sin\alpha_o + a_m]\sin\beta_o + a_c u_c^2 \cos\alpha_o \sin\beta_o + l_c \cos\beta_o \end{bmatrix} \tag{18}
$$

The unit normal of Σ_c is represented as

$$
\mathbf{n}_c = \frac{\dfrac{\partial \mathbf{r}_c}{\partial u_c} \times \dfrac{\partial \mathbf{r}_c}{\partial l_c}}{|\dfrac{\partial \mathbf{r}_c}{\partial u_c} \times \dfrac{\partial \mathbf{r}_c}{\partial l_c}|} \tag{19}
$$

Equations (18) and (19) yield

$$
\mathbf{n}_c = \frac{1}{(1 + 4a_c^2 u_c^2)^{0.5}} \begin{bmatrix} \sin\alpha_o - 2a_c u_c \cos\alpha_o \\ -(\cos\alpha_o + 2a_c u_c \sin\alpha_o)\cos\beta_o \\ (\cos\alpha_o + 2a_c u_c \sin\alpha_o)\sin\beta_o \end{bmatrix} \tag{20}
$$

Using equations (13), (15), (18) and (20), it is easy to verify that surfaces Σ_c and Σ_t are in tangency along the z_b axis when $u_c = u_t = 0$.

4. Pinion and Gear Surfaces Generated by Rack-Cutters

In the process for generation the two rigidly connected rack-cutters perform translational motion while the pinion and the gear perform rotational motions as shown in fig. 6. To provide a predesigned parabolic function of transmission errors for each cycle of meshing, it is necessary to observe certain relations between the motions of the rack-cutters and gears, respectively.

The angle ψ_{pc} of pinion rotation and the displacement s_c of rack-cutter Σ_c are related by the following linear function

$$\psi_{pc} = \frac{s_c}{r_p} \tag{21}$$

Here: r_p is the radius of the pinion pitch cylinder.

The angle ψ_{gt} of gear rotation and the displacement s_t of rack-cutter Σ_t are related as follows

$$\psi_{gt} = \frac{N_p}{N_g}\left(\frac{s_t}{r_p}\right) - a\left(\frac{s_t}{r_p} - \psi^{(0)}\right)^2 \tag{22}$$

Here: N_p and N_g are the tooth numbers of the pinion and gear, respectively, and $\psi^{(0)}$ is the initial position angle of the gear for the modification gear rotation.

Equation of Meshing between Rack-Cutter Σ_c and Pinion Σ_p

The equation of meshing between rack-cutter Σ_c and the pinion tooth surface Σ_p is represented as

$$f(u_c,\ \ l_c,\ \ \psi_{pc}) = \mathbf{N}_c^{(c)} \cdot \mathbf{v}_c^{(cp)} = 0 \tag{23}$$

where ψ_{pc} is the angle of rotation of the pinion in the process for generation. The normal $\mathbf{N}_c^{(c)}$ to Σ_c in S_c can be obtained by equation (20), and the relative velocity of the pinion with respect to Σ_c may be represented as

$$\mathbf{v}_c^{(pc)} = \boldsymbol{\omega}_c^{(p)} \times (\mathbf{R}_p + \mathbf{r}_c) - (\ 0 \quad r_p \quad 0\ )^T \frac{d\psi_{pc}}{dt} \tag{24}$$

Here: $\mathbf{R}_p = (\overline{O_p O_c})_c = (r_p \quad r_p\psi_{pc} \quad 0)^T$, $r_p = E_{pc}$ is the radius of the pitch cylinder of the pinion (fig. 6), $\boldsymbol{\omega}_c^{(p)} = \omega_c^{(p)}(\ 0\ \ 0\ \ 1\ )^T$.

Substitution of equations (19) and (24) into (23), yields the following equation of meshing between Σ_p and Σ_c

$$f(u_c, l_c, \psi_{pc}) = l_c \sin\beta_o + r_p\psi_{pc} + a_m \cos\beta_o + \frac{[(u_c - d_p) + 2a_c^2 u_c^3]\cos\beta_o}{\sin\alpha_o - 2a_c u_c \cos\alpha_o} = 0 \tag{25}$$

Surface of Pinion Σ_p

In the process of generation of pinion surface, rack-cutter Σ_c performs uniform translation and the workpiece performs uniform rotation (fig. 6(b)). The transformation matrix from system S_c to S_p can be represented as

$$\boldsymbol{M}_{pc} = \begin{bmatrix} \cos\psi_{pc} & \sin\psi_{pc} & 0 & r_p\cos\psi_{pc} + r_p\psi_{pc}\sin\psi_{pc} \\ -\sin\psi_{pc} & \cos\psi_{pc} & 0 & -r_p\sin\psi_{pc} + r_p\psi_{pc}\cos\psi_{pc} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{26}$$

Pinion surface Σ_p in system S_p is represented as

$$\left. \begin{array}{c} \mathbf{r}_p(u_c, l_c, \psi_{pc}) = \mathbf{M}_{pc}\mathbf{r}_c \\[2ex] l_c = -\{\dfrac{(u_c - d_p) + 2a_c^2 u_c^3}{\sin\alpha_o - 2a_c u_c \cos\alpha_o}\cos\beta_o + r_p\psi_{pc} + a_m\cos\beta_o\}/\sin\beta_o \end{array} \right\} \tag{27}$$

Substituting equation (26) into (27), we obtain equation of Σ_p as

$$\boldsymbol{r}_p(u_c, \psi_{pc}) = \boldsymbol{r}_p(u_p, \theta_p) \tag{28}$$

Equation of Meshing between Rack-Cutter Σ_t and Gear Σ_g

The equation of meshing between rack-cutter Σ_t and the gear tooth surface Σ_g is represented as

$$f(u_t, \quad l_t, \quad \psi_{gt}) = \mathbf{n}_t^{(t)} \cdot \mathbf{v}_t^{(gt)} = 0 \tag{29}$$

where ψ_{gt} is the angle of rotation of the gear in the process for generation. The unit normal $\mathbf{n}_t^{(t)}$ to Σ_t in S_t is represented by equation (15), and $\mathbf{v}_t^{(gt)}$ is the relative velocity of the gear with respect to rack-cutter Σ_t.

We recall that the rack-cutter Σ_t performs translation with constant velocity, but the gear performs rotation with variable angular velocity that is represented as (see equation (22))

$$\boldsymbol{\omega}_t^{(g)} = \begin{bmatrix} 0 & 0 & -[\frac{N_p}{N_g} - 2a(\psi_{pc} - \psi^{(0)})] \end{bmatrix}^T \frac{d\psi_{pc}}{dt} \tag{30}$$

The relative velocity $\mathbf{v}_t^{(gt)}$ is represented as

$$\mathbf{v}_t^{(gt)} = \boldsymbol{\omega}_t^{(g)} \times (\mathbf{R}_g + \mathbf{r}_t) - (\ 0 \quad \frac{N_p}{N_g} r_g \quad 0\ )^T \frac{d\psi_{pc}}{dt} \tag{31}$$

where

$$\mathbf{R}_g = \begin{pmatrix} -r_g & \frac{N_p}{N_g} r_g \psi_{pc} & 0 \end{pmatrix}^T \tag{32}$$

and $r_g = E_{gt}$ is the radius of pitch cylinder of the gear (fig. 6).

Substitution of equations (15), (30), (31) and (32) into (29) yields the following equation of meshing between Σ_g and Σ_t

$$\begin{aligned} f(u_t, l_t, \psi_{gt}) &= (u_t - d_p)\cos\beta_o + \sin\alpha_o(l_t \sin\beta_o + \frac{N_p}{N_g} r_g \psi_{pc} + a_m \cos\beta_o) \\ &+ \frac{2aN_g r_g(\psi_{pc} - \psi^{(0)})}{N_p - 2aN_g(\psi_{pc} - \psi^{(0)})}\cos\alpha_o \cos\beta_o \end{aligned} \tag{33}$$

where

$$\psi_{gt} = \frac{N_p}{N_g}\psi_{pc} - a(\psi_{pc} - \psi^{(0)})^2 \tag{34}$$

Surface of Gear Σ_g

It must be remembered that the gear with the tooth surface Σ_g performs rotation about its axis with varied angular velocity while rack-cutter Σ_t performs uniform translation (fig. 6(a)). The transformation matrix from system S_t to S_g can be represented as

$$
\boldsymbol{M}_{gt} = \begin{bmatrix} -\cos\psi_{gt} & \sin\psi_{gt} & 0 & r_g\cos\psi_{gt} + r_g\dfrac{N_p}{N_g}\psi_{pc}\sin\psi_{gt} \\ -\sin\psi_{gt} & -\cos\psi_{gt} & 0 & r_g\sin\psi_{gt} - r_g\dfrac{N_p}{N_g}\psi_{pc}\cos\psi_{gt} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{35}
$$

Gear tooth surface Σ_g in system S_g can be represented as

$$
\left.\begin{aligned} &\mathbf{r}_g(u_t, l_t, \psi_{gt}) = \mathbf{M}_{gt}\mathbf{r}_t \\ l_t = -\{(u_t - d_p)&\frac{\cot\beta_o}{\sin\alpha_o} + (\frac{N_p}{N_g}r_g\psi_{pc} + a_m\cos\beta_o)\frac{1}{\sin\beta_o} \\ &+\frac{2aN_g r_g(\psi_{pc} - \psi^{(0)})}{N_p - 2aN_g(\psi_{pc} - \psi^{(0)})}\cot\alpha_o\cot\beta_o\} \end{aligned}\right\} \tag{36}
$$

The derivation of equation (36) is based on transformation of equations (33) and (34). Equations (35) and (36) enable to represent the gear tooth surface in two-parameter form as follows

$$
\boldsymbol{r}_g(u_t, \psi_{gt}) = \boldsymbol{r}_g(u_g, \theta_g) \tag{37}
$$

5. Computerized Simulation of Meshing and Contact of Pinion-Gear Tooth Surfaces

We consider that the surfaces of the pinion and the gear generated by worms Σ_w and Σ_h are represented in coordinate systems S_p and S_g, respectively. The fixed coordinate system S_f

is rigidly connected to the housing of the gear drive (figs. 7 and 8). The movable coordinate systems S_p and S_g are rigidly connected to the pinion and the gear, respectively. An auxiliary coordinate system S_h is applied for simulation of meshing when the gear axis is crossed or intersected with the pinion axis instead of being parallel, and when the shortest distance between the pinion and gear axes is changed. The errors of misalignment are referred to the gear. The misalignment angle $\Delta\gamma$ is decomposed into two components, $\Delta\gamma_x$ and $\Delta\gamma_y$ that represent the crossing angle and the intersection angle, respectively. The pinion performs rotational motion about the z_f-axis. The axis of gear rotation is z_h. The shortest distance between the axes of rotation is designated as E_{pg}.

The rotation matrices from system S_h to S_f for crossed and intersecting angles are represented in the followings (fig. 8)

$$\boldsymbol{L}_{fh} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\Delta\gamma_x & \sin\Delta\gamma_x \\ 0 & \sin\Delta\gamma_x & \cos\Delta\gamma_x \end{bmatrix} \tag{38}$$

$$\boldsymbol{L}_{fh} = \begin{bmatrix} -\cos\Delta\gamma_y & 0 & -\sin\Delta\gamma_y \\ 0 & -1 & 0 \\ -\sin\Delta\gamma_y & 0 & \cos\Delta\gamma_y \end{bmatrix} \tag{39}$$

We represent the pinion and gear tooth surfaces, Σ_p and Σ_g, and their unit normals in coordinate system S_f. The conditions of continuous tangency of surfaces Σ_p and Σ_g are represented by the following equations [1,2].

$$\boldsymbol{r}_f^{(p)}(u_p, \theta_p, \phi_p) = \boldsymbol{r}_f^{(g)}(u_g, \theta_g, \phi_g) \tag{40}$$

$$\boldsymbol{n}_f^{(p)}(u_p, \theta_p, \phi_p) = \boldsymbol{n}_f^{(g)}(u_g, \theta_g, \phi_g) \tag{41}$$

Vector equation (41) provides only two independent equations since $|\mathbf{n}_f^{(p)}| = |\mathbf{n}_f^{(g)}| = 1$. The total number of independent equations provided by (40) and (41) is five that relate six parameters

$$f_i(u_p, \theta_p, \phi_p, u_g, \theta_g, \phi_g) = 0 \qquad (i = 1, 2, \dots 5) \tag{42}$$

The continuous solution of the system of nonlinear equations (42) is based on the following procedure:

(1) Using an initial guess, we determine a set of parameters that satisfy equation system (42). Thus

$$P^{(0)} = (u_p^{(0)}, \theta_p^{(0)}, u_g^{(0)}, \theta_g^{(0)}, \phi_g^{(0)}) \tag{43}$$

(2) One of the variable parameters, say ϕ_p, is chosen as the input one, and is supposed that the Jacobian

$$\frac{D(f_1, f_2, f_3, f_4, f_5)}{D(u_p, \theta_p, u_g, \theta_g, \phi_g)} \tag{44}$$

differs from zero. The derivatives in the Jacobian are taken at point $P^{(0)}$.

(3) Then, equation system (42) can be solved in the neighborhood of $P^{(0)}$ by functions

$$\phi_g(\phi_p), u_p(\phi_p), \theta_p(\phi_p), u_g(\phi_p), \theta_g(\phi_p) \tag{45}$$

(4) Vector function $\mathbf{r}_p(u_p, \theta_p)$ that determines the pinion surface Σ_p and functions $u_p(\phi_p)$, $\theta_p(\phi_p)$ enable to determine the path of contact on Σ_p.

(5) Similarly, we can obtain the path of contact on the gear surface Σ_g using vector function $\mathbf{r}_g(u_g, \theta_g)$ and functions $u_g(\phi_p)$, $\theta_g(\phi_g)$.

(6) The paths of contact on pinion and gear tooth surfaces slightly deviate from helices in the case of an aligned gear drive. The line of action for an aligned gear drive (the set of contact points in S_f) slightly deviates from a straight line that is parallel to the gear axes.

(7) The transmission function $\phi_g(\phi_p)$ deviates from the ideal transmission function, and the function of transmission errors coincides with the predesigned parabolic function.

(8) The determination of dimensions and orientation of the instantaneous contact ellipse needs the knowledge of the principal curvatures and directions of contacting surfaces and the elastic approach of surfaces. This problem can be substantially simplified if the pinion-gear principal curvatures and directions are expressed in terms of the principal curvatures and directions of the generating surfaces and parameters of motion [1,2].

6. Numerical Example

The method developed in this report is illustrated with the example discussed below. The design parameters of the pinion and gear are listed in Table 1. The numerical simulation of meshing is performed for an aligned and misaligned gear drives with various errors of alignment for the pinion and gear.

Case 1. Aligned gear drive

Figure 9 shows the transmission errors for the aligned gear drive. The TCA performed

Table 1: Design parameters of pinion and gear

	pinion	gear
tooth number	$N_p = 20$	$N_g = 100$
normal diametral pitch	$P_n = 5\ \frac{1}{in}$	$P_n = 5\ \frac{1}{in}$
normal pressure angle	$\alpha_o = 20^o$	$\alpha_o = 20^o$
helix angle on pitch cylinder	$\beta_o = 30^o$	$\beta_o = 30^o$
tooth length	L=1.6 in.	L=1.6 in.
modification coefficient	$a_c = 0.0008$	$a = 0.0014$
elastic approach	$\delta = 0.007\ mm.$	$\delta = 0.007\ mm.$

confirms that the predesigned parabolic function of transmission errors exists. The maximum transmission error is approximately 8 arc seconds. Figure 10 shows the contact pattern and the contact path. The path of contact on the tooth surface is in the longitudinal direction. The major axis of the instantaneous contact ellipse is 6 mm for the assumed elastic approach of the surfaces equal to 0.007 mm.

Case 2. The pinion-gear rotation axes are crossed

Figures 11 and 12 show the transmission errors and the contact pattern for the case when the crossing angle $\Delta\gamma_{cross}$ is 4 arc minutes. The maximum transmission error is 8 arc seconds and contact paths are shifted up and down on the gear and pinion surfaces, respectively. Figures 13 and 14 show the transmission errors and contact pattern for $\Delta\gamma_{cross} = -4$ arc minutes.

Case 3. The pinion-gear rotation axes are intersected

Figures 15 and 16 show the transmission errors for the misalignment $\Delta\gamma_{intersect} = 4$ arc minutes. Figures 17 and 18 show the transmission errors and contact pattern for the mentioned above error of alignment.

Case 4. Influence of error of lead angle

Figures 19 and 20 show the transmission errors and contact pattern when the error of the lead angle is 4 arc minutes.

Figures 21 and 22 show the transmission errors and contact pattern when the error of the lead angle is -4 arc minutes.

For all of above cases, the maximum transmission error does not exceed 8 arc second (with very small deviations of this value).

7. Conclusion

From the analytical study presented in this report the following conclusions can be drawn:

(1) The interaction of a parabolic and a linear functions of transmission errors has been discussed to prove the possibility to absorb almost, linear functions of transmission errors caused by misalignment.

(2) Mismatched surfaces of two rack-cutters for generation of modified involute gears have been proposed.

(3) Generation and geometry of pinion-gear modified tooth surfaces have been determined.

(4) Computerized simulation of meshing and contact of pinion-gear tooth surfaces has been developed.

(5) An algorithm for determination of relations between the curvatures of the generating and the generated surfaces has been developed.

(6) An algorithm for determination of the contact ellipse has been developed.

(7) Directions for users of application of developed computer programs for the design of gears with the modified geometry and computerized simulation of their meshing and contact have been developed (Appendix C).

References

1. Litvin, F.L.: "Theory of Gearing", NASA Publication 1212, 1989.

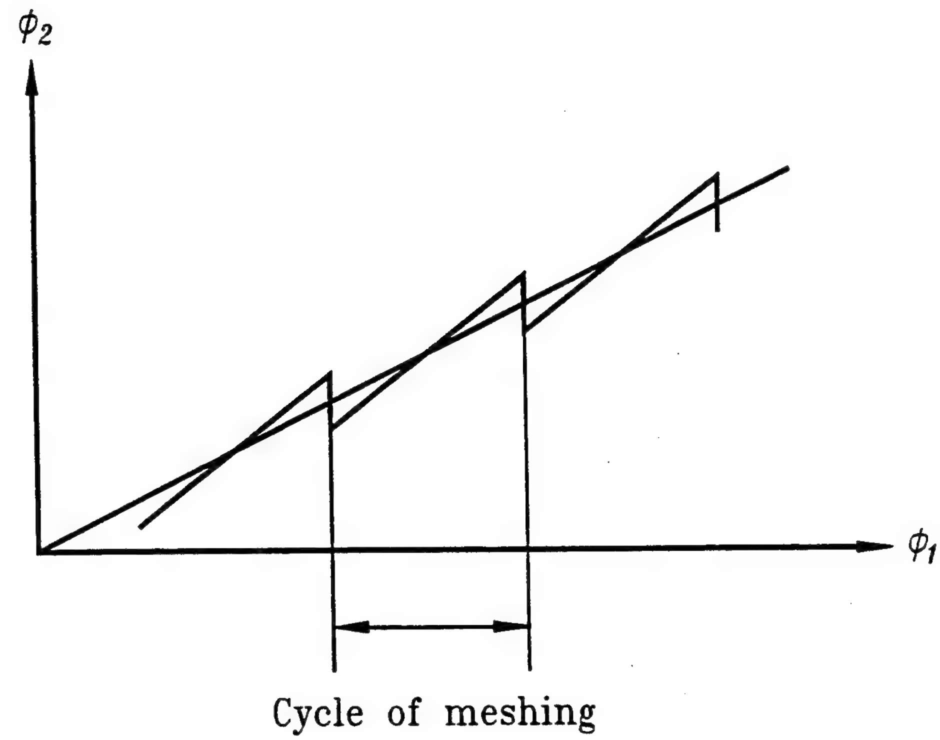
2. Litvin, F.L.: "Gear Geometry and Applied Theory", Prentice Hall, Englewood Cliffs, New Jersey, 1994.

3. Litvin, F.L. and Krylov, N.N. and Erichov, M.L.: "Generation of Tooth Surfaces by Two-Parameter Enveloping", *Mechanism and Machine Theory*, 1975, Vol.10, pp. 365-373.

4. Litvin, F.L. and Zhang, J., Handschuh, R.F. and Coy, J.J.: "Topology of Modified Helical Gears", *Surface Topology*, p.p. 41-58, March, 1989.

5. Reishauer CNC Gear Grinding Machines, Catalogs, Switzerland.

(a)



(b)

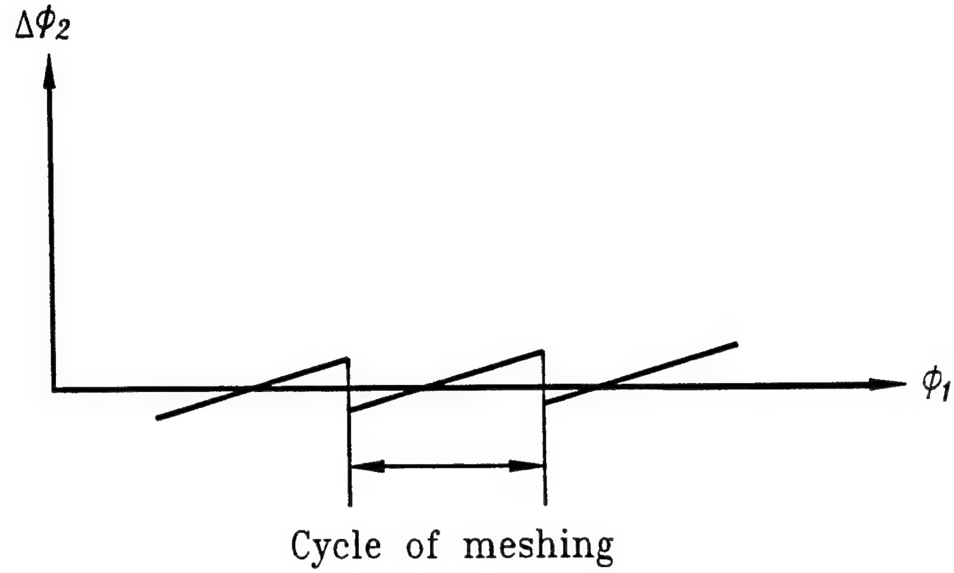


Fig. 1 Transmission function and transmission errors for a misaligned gear drive

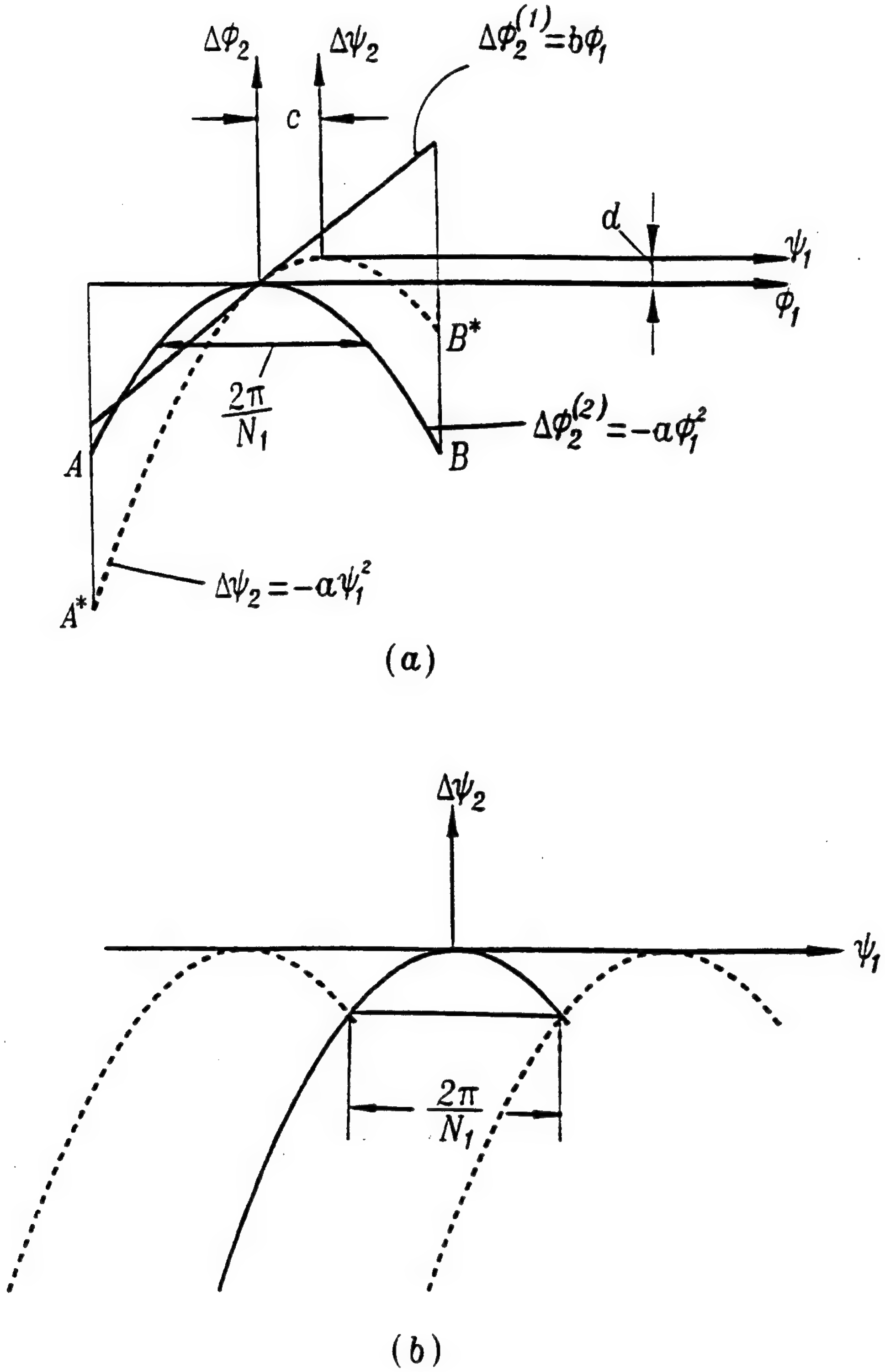


Fig. 2 Interaction of parabolic and linear functions

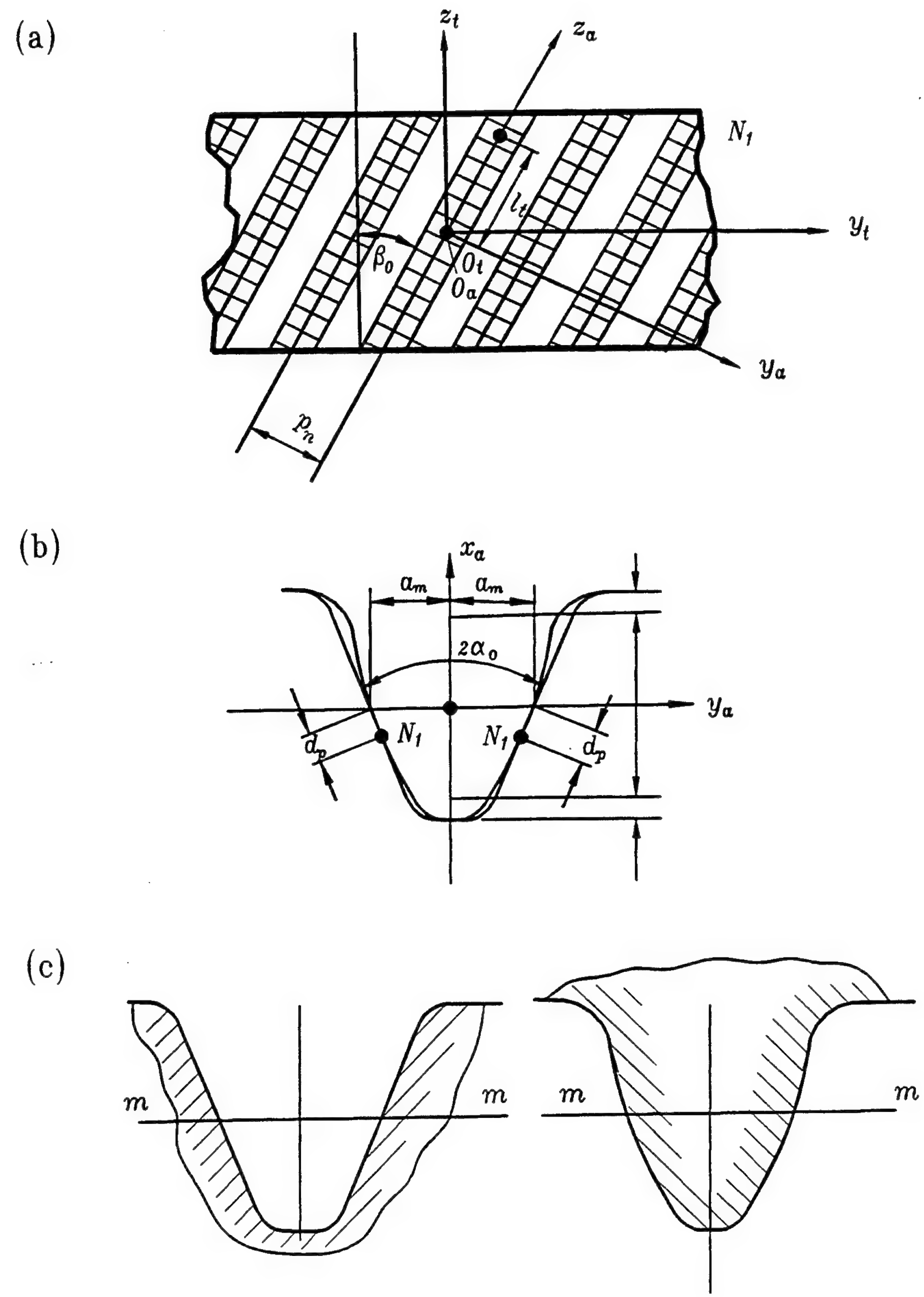


Fig. 3 Normal sections of rack–cutters

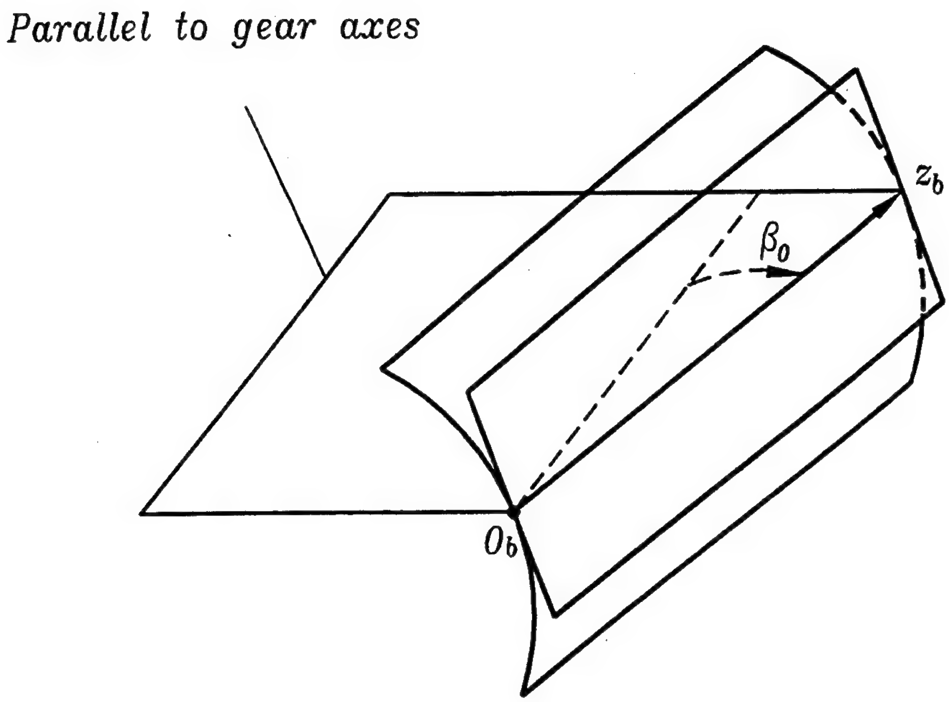


Fig. 4 Orientation of rack-cutters with respect to gear axes

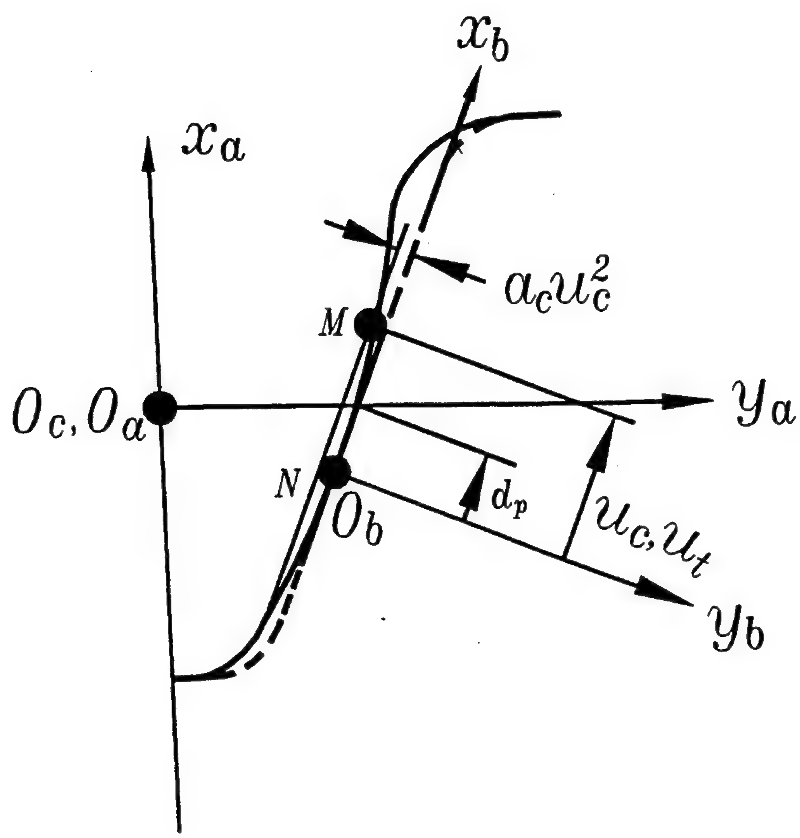


Fig. 5 Normal section of pinion rack-cutter surface

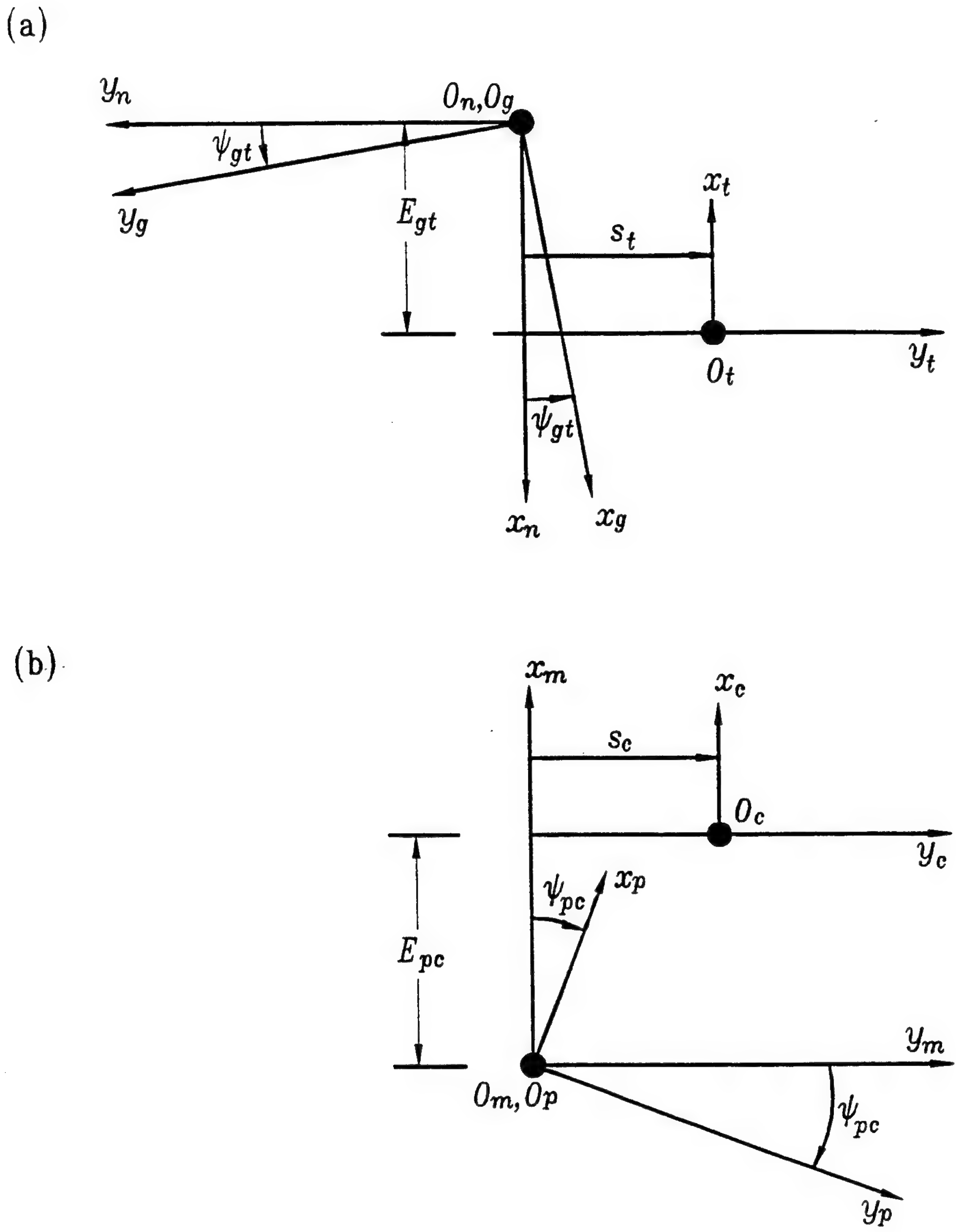


Fig. 6 Generation of pinion and gear by rack–cutters

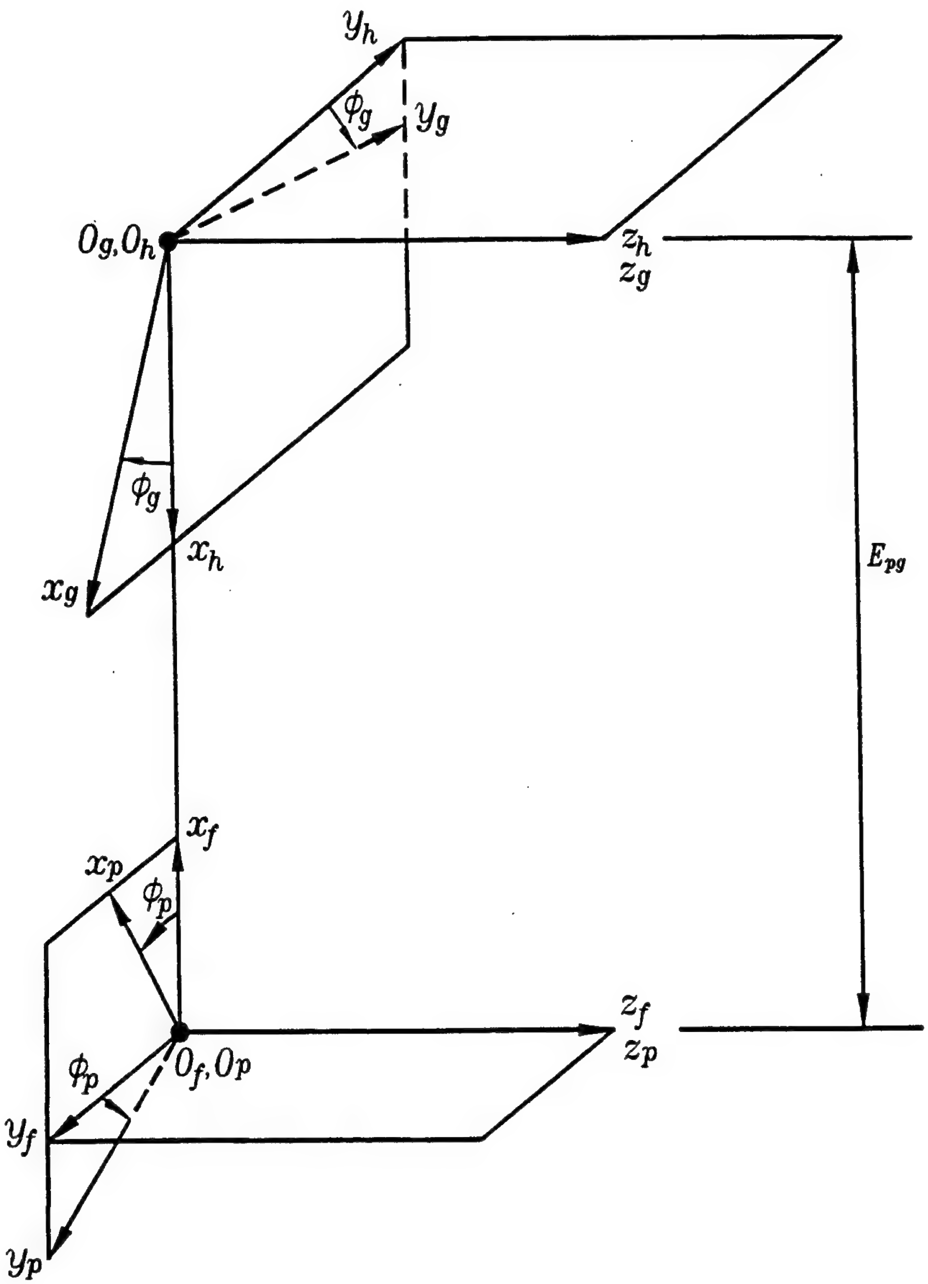


Fig. 7 Coordinate system applied for tooth contact analysis (TCA)

(a)

(b)

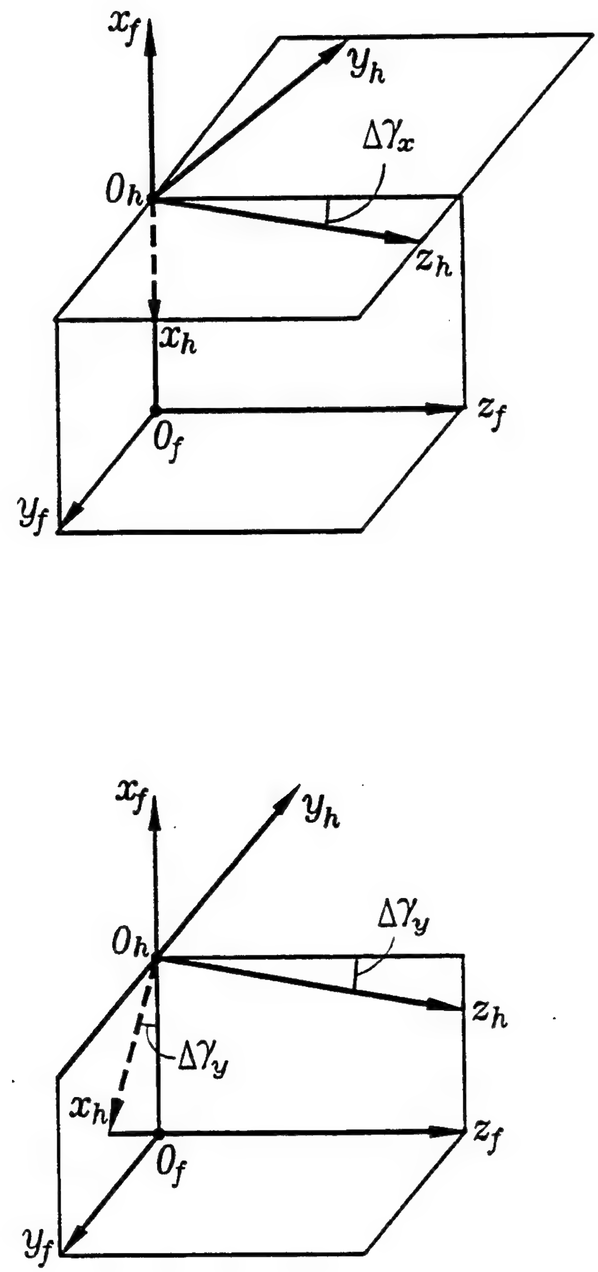


Fig. 8 Coordinate systems S_h and S_f

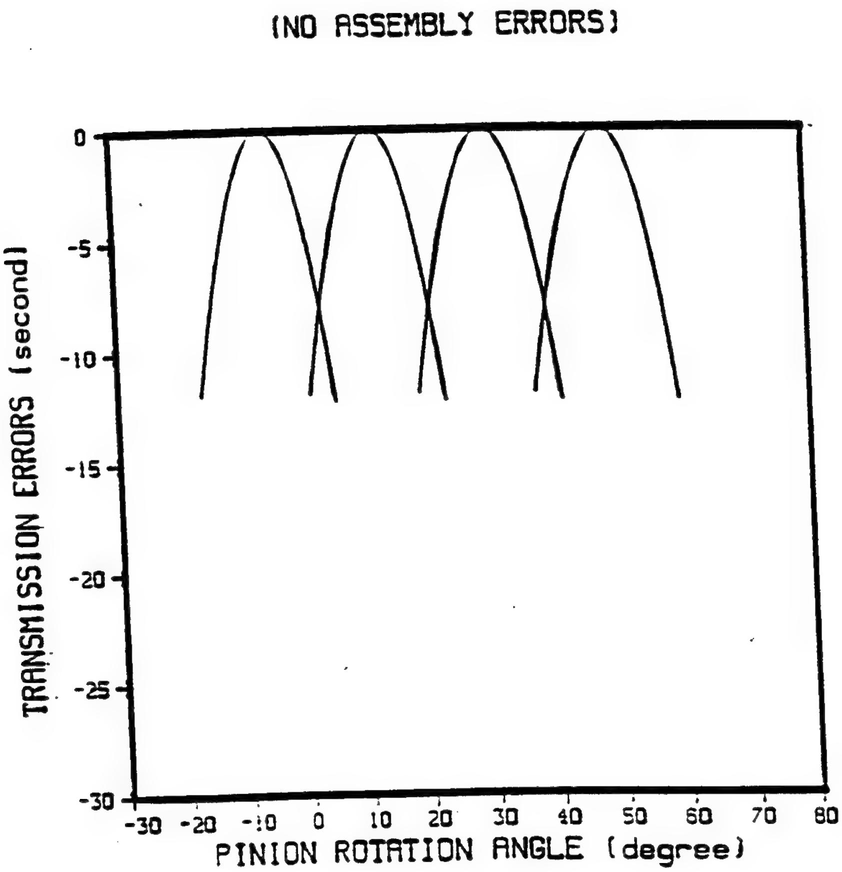


Fig. 9 Transmission errors for the aligned gear drive

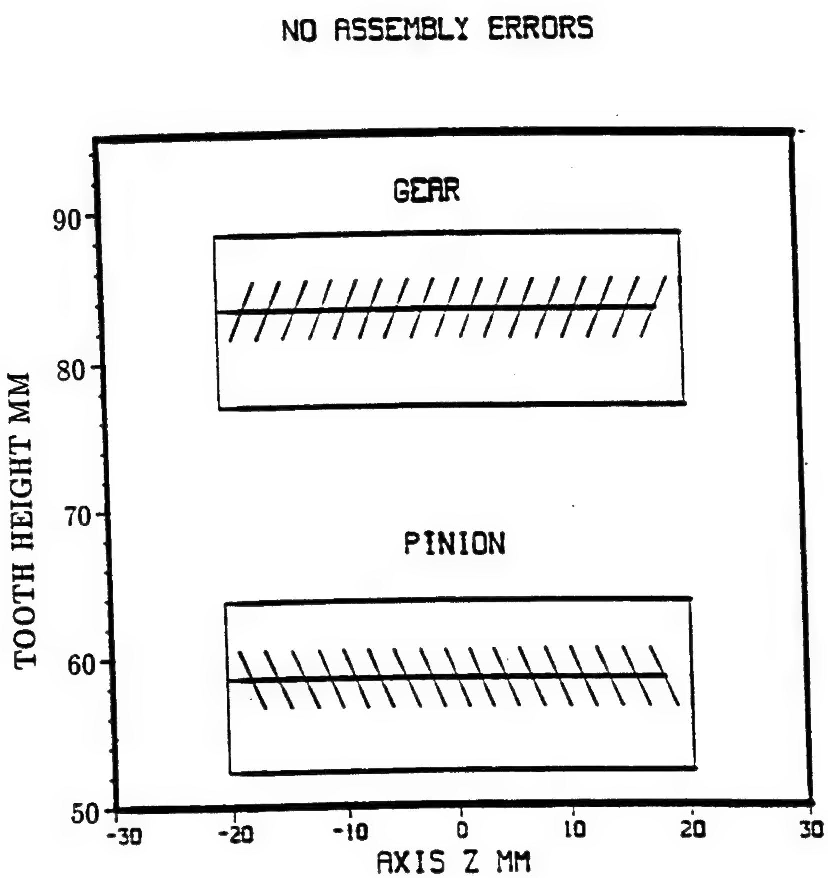


Fig. 10 Contact paths and pattern for the aligned gear drive

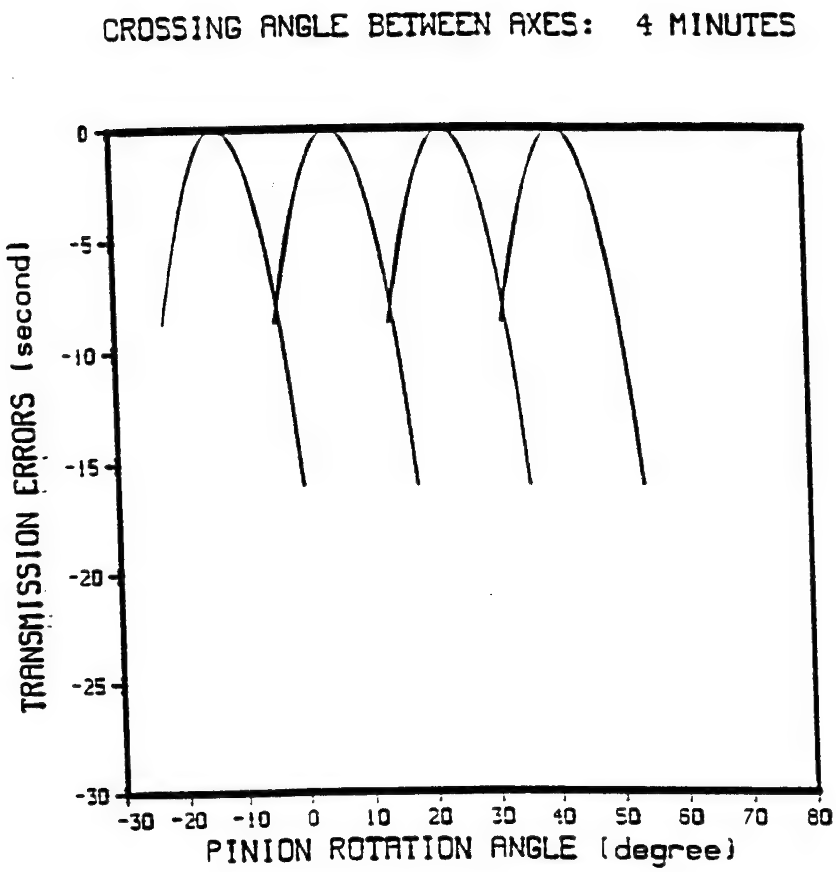


Fig. 11 Transmission errors for the alignment error $\Delta\gamma_{cross}$ = 4 arc minutes

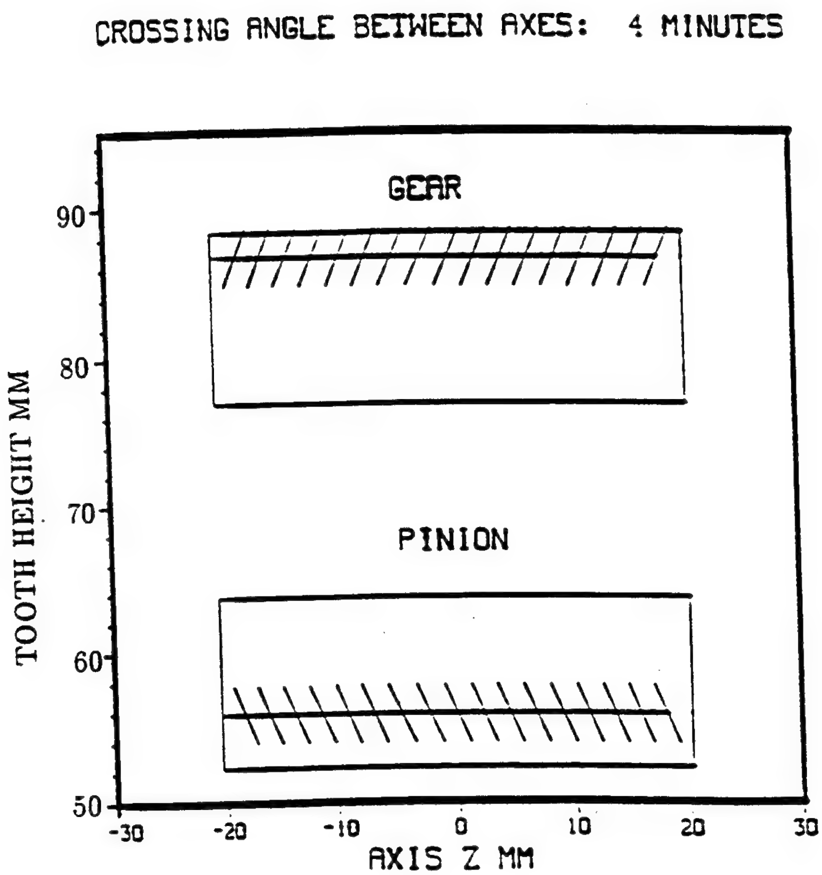


Fig. 12 Contact pattern for the alignment error

$\Delta\gamma_{cross} = 4$ arc minutes

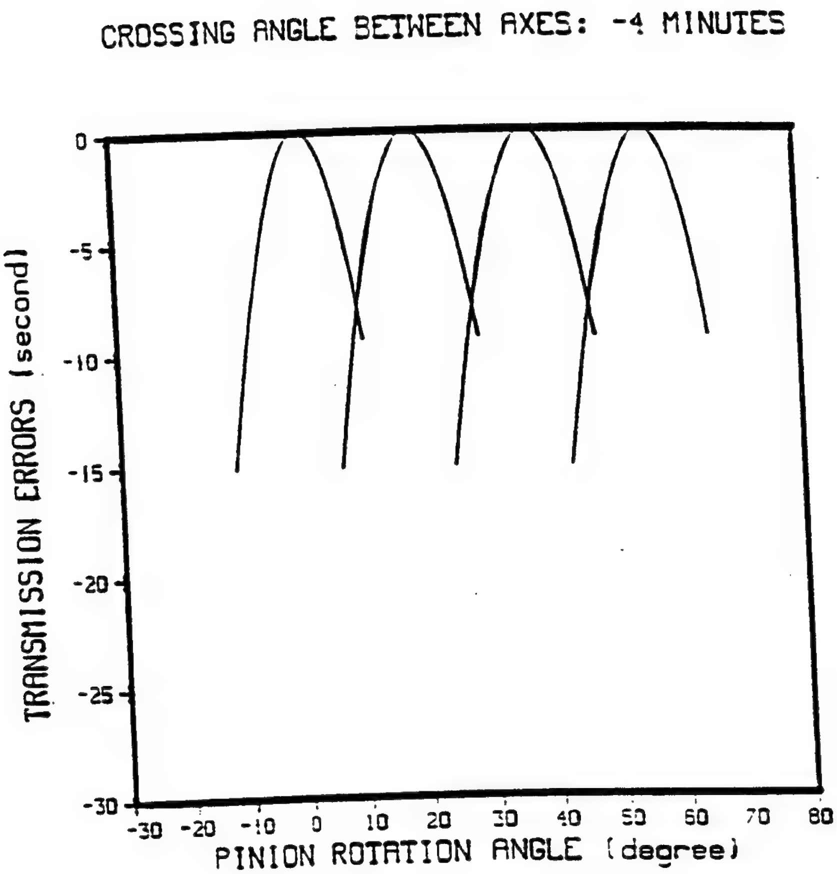


Fig. 13 Transmission errors for the alignment error

$\Delta\gamma_{cross} = -4$ arc minutes

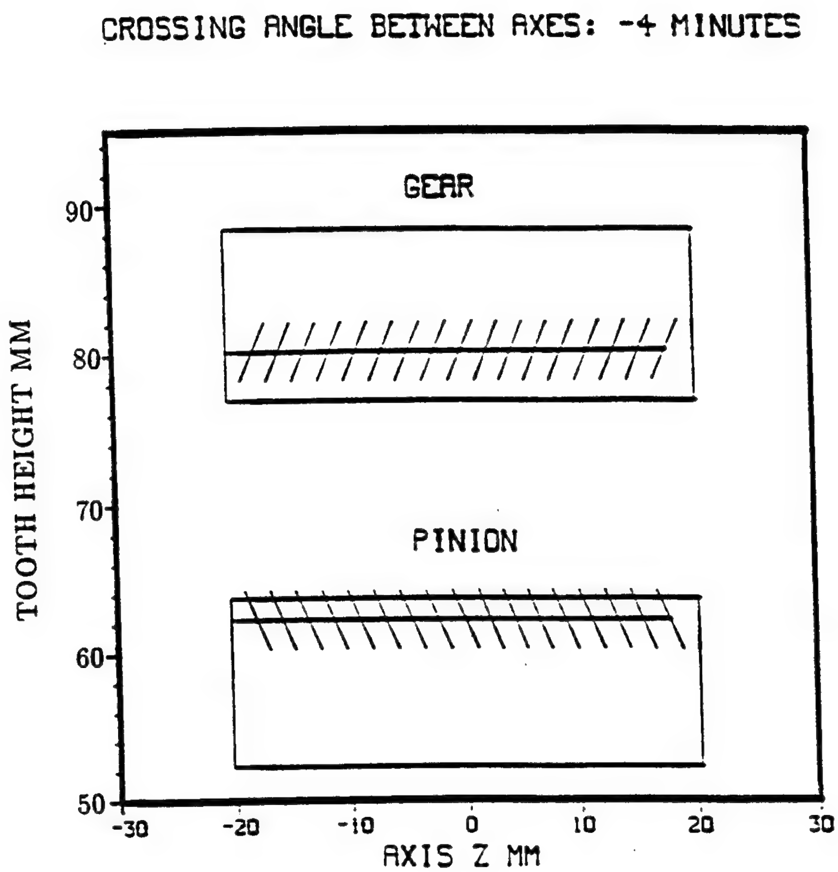


Fig. 14 Contact pattern for the alignment error

$\Delta\gamma_{cross} = -4$ arc minutes

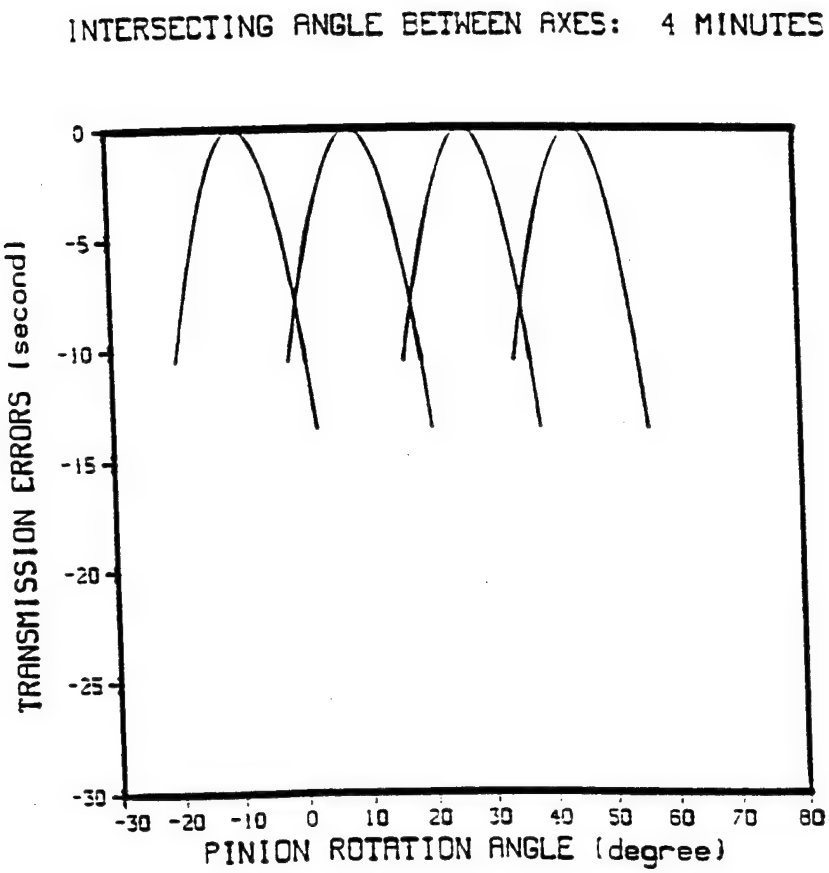


Fig. 15 Transmission errors for misaligned gear drive with $\Delta\gamma_{intersectting}$ = 4 arc minutes

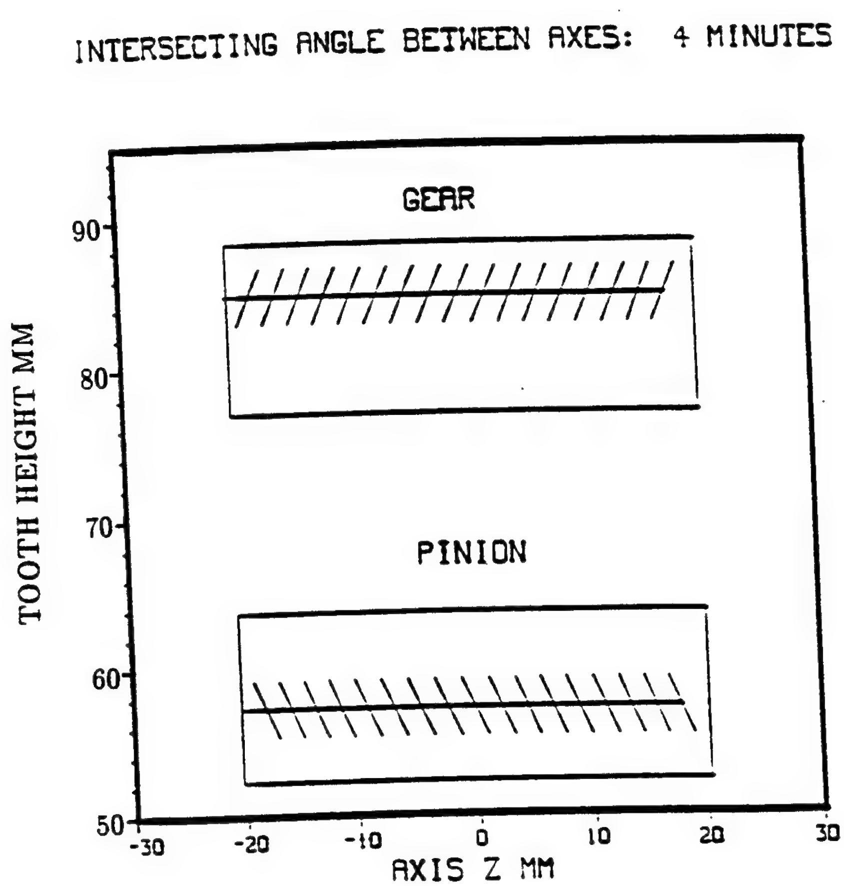


Fig. 16 Contact pattern and paths for the misaligned gear drive with $\Delta\gamma_{intersectting}$ = 4 arc minutes

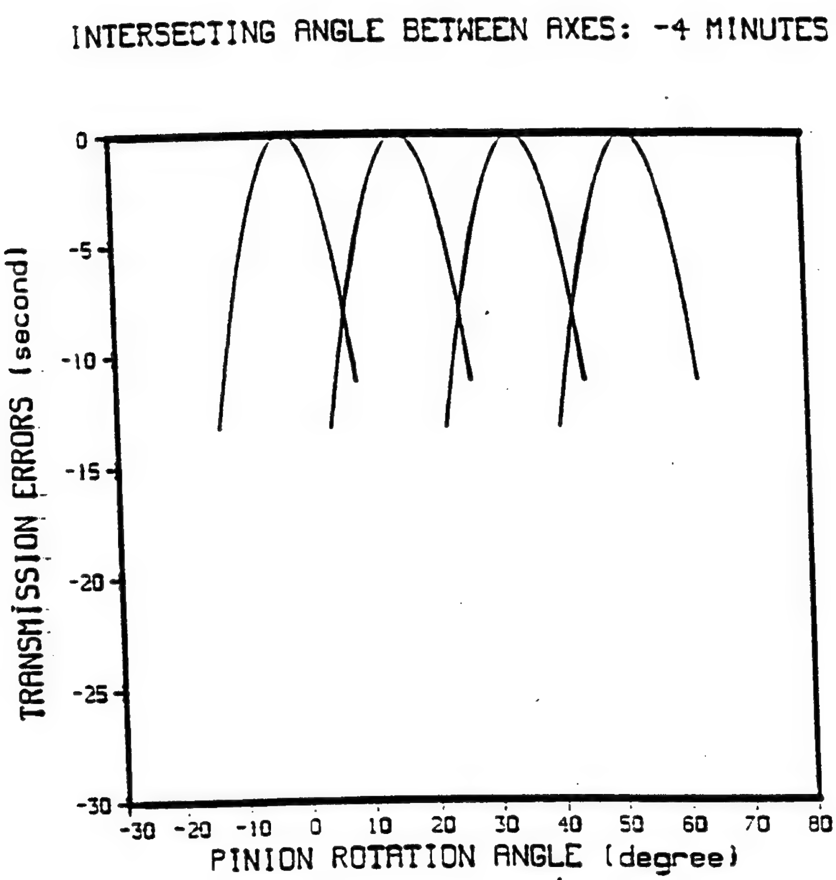


Fig. 17 Transmission errors for the misaligned gear drive with $\Delta\gamma_{intersectting} = -4$ arc minutes

INTERSECTING ANGLE BETWEEN AXES: -4 MINUTES

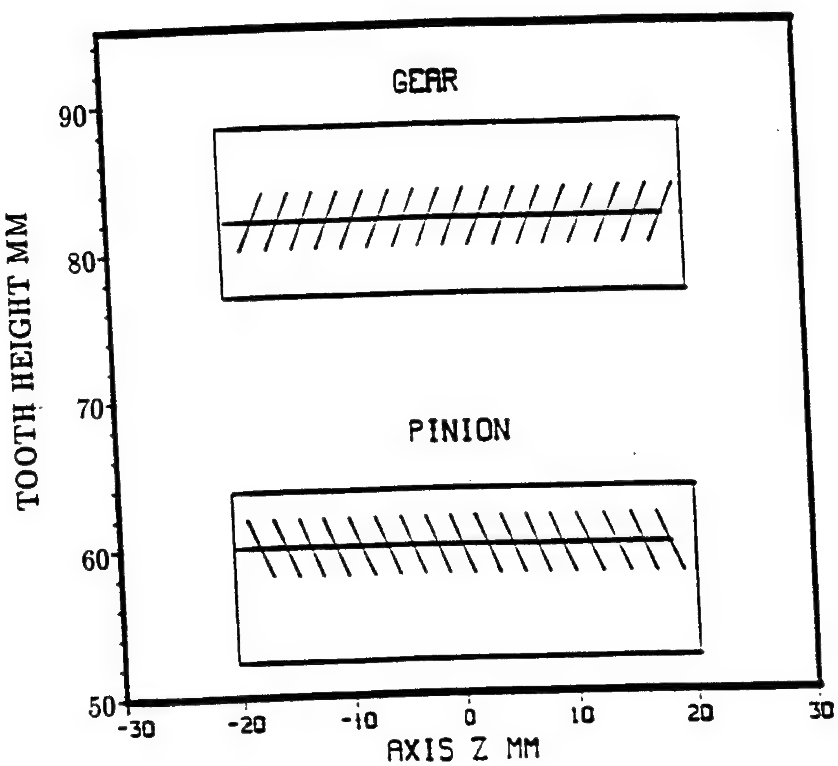


Fig. 18 Contact pattern for the misaligned gear drive with $\Delta\gamma_{intersectting} = -4$ arc minutes

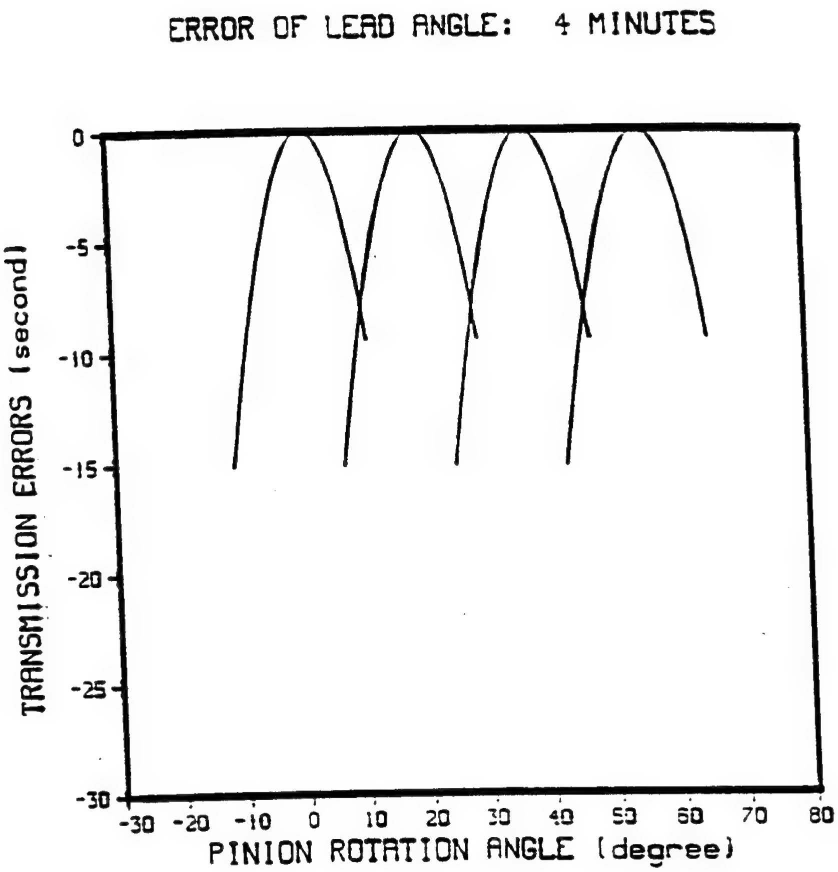


Fig. 19 Transmission errors for the misaligned gear drive with $\Delta\lambda_0 = 4$ arc minutes

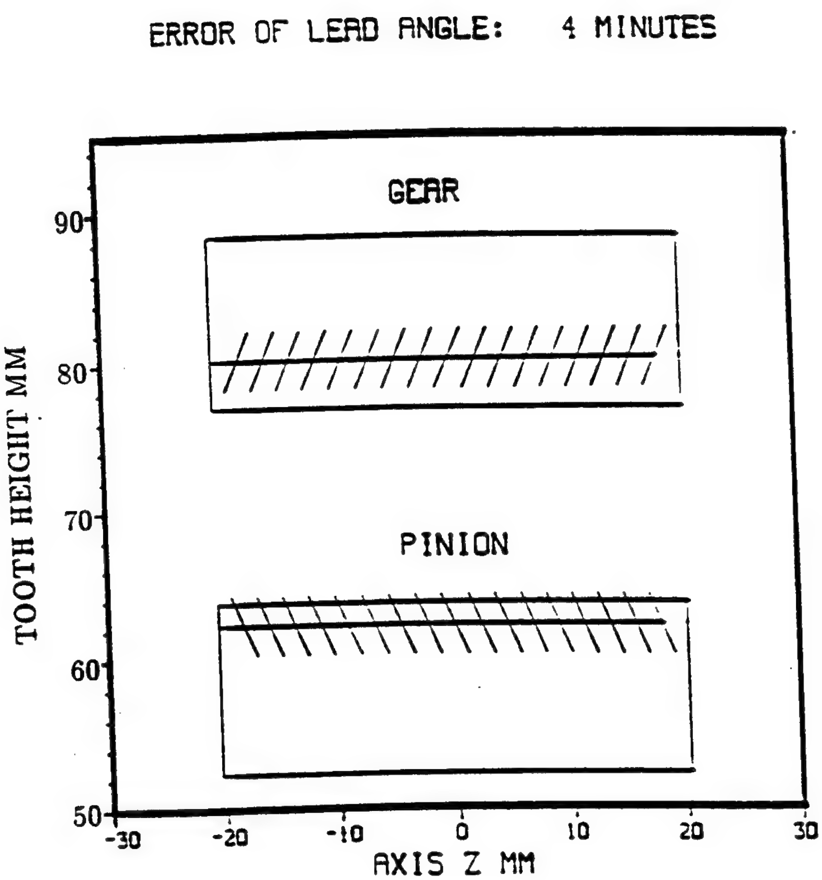


Fig. 20 Contact pattern and contact path for misaligned gear drive with $\Delta\lambda_0 = 4$ arc minutes

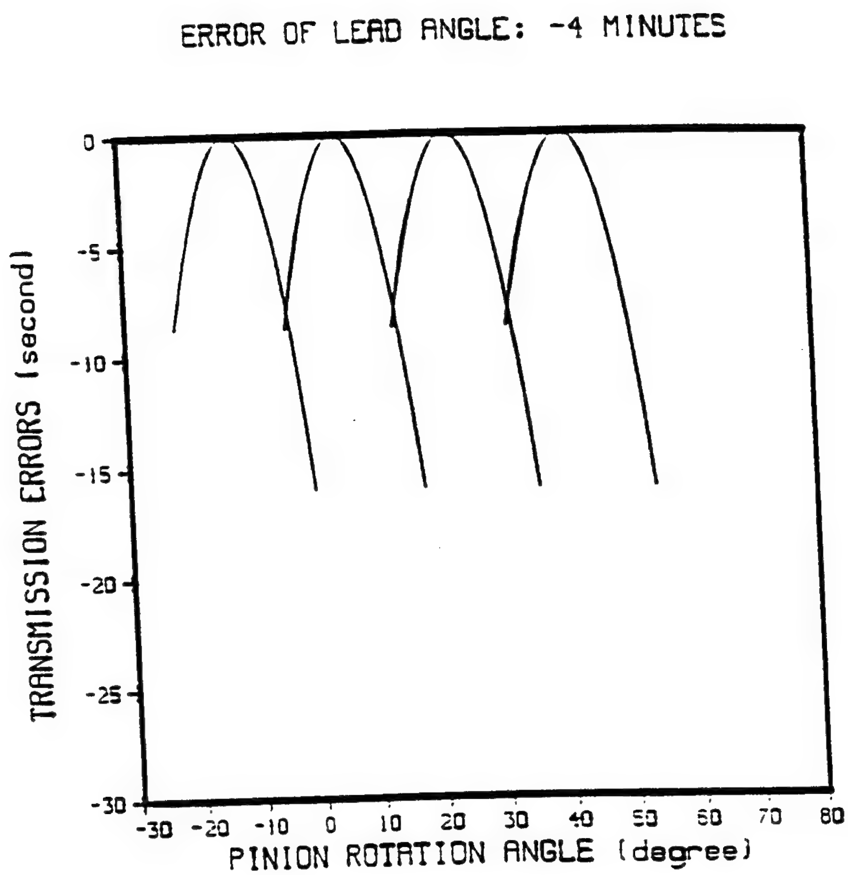


Fig. 21 Transmission errors for the misaligned gear drive with $\Delta\lambda_0 = -4$ arc minutes

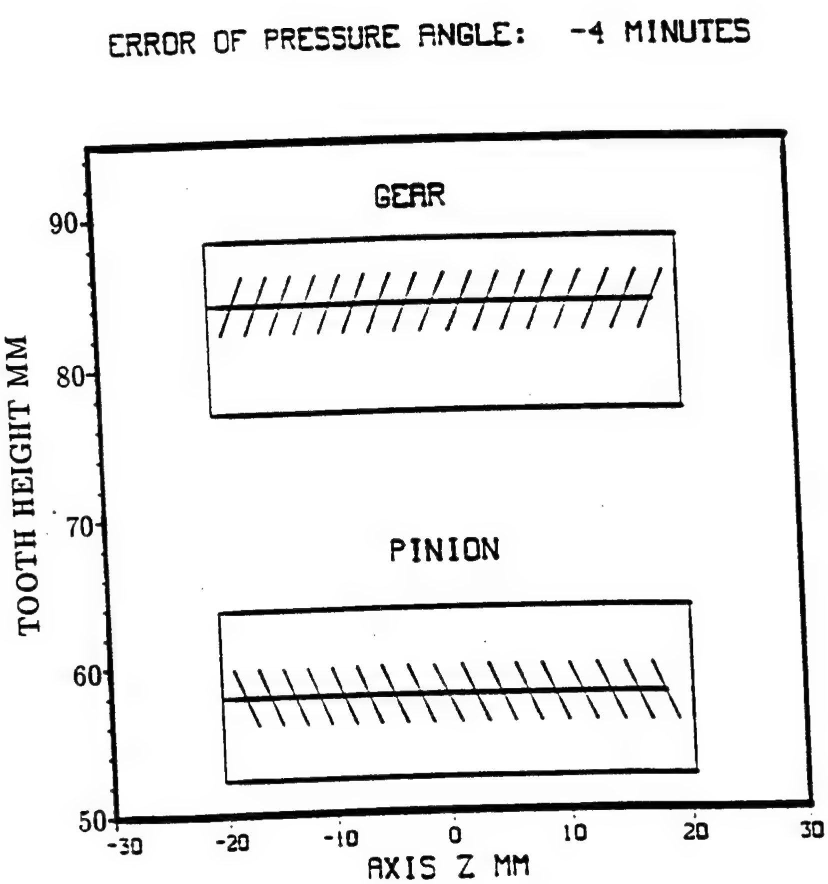


Fig. 22 Contact pattern and contact paths for the misaligned gear drive with $\Delta\lambda_0 = -4$ arc minutes

Appendix A. Relations between the Curvatures of the Generating and Generated Surfaces

Direct Relations between Principal Curvatures and Directions of Mating Surfaces

The main advantage of this approach (proposed by Litvin) is the possibility to determine the principal curvatures and directions of the generated surface in terms of principal curvatures and directions of the generating tool surface, and the parameters of motion. In this case, the tool surfaces for the generation of the gear and the pinion tooth surfaces are rack cutters. The equations developed permit a simplified computational procedure.

The system of equations that relate the principal curvatures and directions of the generating and generated surfaces is as follows. Consider that unit vectors $\mathbf{e}_f$ and $\mathbf{e}_h$ represent the principal directions on the tool surface Σ_1 at point P of tangency of surfaces Σ_1 and Σ_2 (fig. A1). The principal curvatures on the mating surfaces κ_f and κ_h of the tool are given; the parameters of motion (see below) are also given.

The goal is to determine angle σ that is formed by unit vectors $\mathbf{e}_f$ and $\mathbf{e}_s$, and principal curvatures κ_s and κ_q. (The unit vectors $\mathbf{e}_s$ and $\mathbf{e}_q$ represent the principal directions on surface Σ_2). The system of equations for determination of σ, κ_s, and κ_q is as follows.

$$\tan 2\sigma = \frac{2b_{15}b_{25}}{b_{25}^2 - b_{15}^2 - (\kappa_f - \kappa_h)t_{33}} \tag{A1}$$

$$\kappa_q - \kappa_s = \frac{2b_{15}b_{25}}{t_{33}\sin 2\sigma} = \frac{b_{25}^2 - b_{15}^2 - (\kappa_f - \kappa_h)t_{33}}{t_{33}\cos 2\sigma} \tag{A2}$$

$$\kappa_q + \kappa_s = \kappa_f + \kappa_h + \frac{b_{15}^2 + b_{25}^2}{t_{33}} \tag{A3}$$

Here:

$$b_{15} = -(\boldsymbol{\omega}^{(12)} \cdot \mathbf{e}_h) - \kappa_f(\mathbf{v}^{(12)} \cdot \mathbf{e}_f) \tag{A4}$$

$$b_{25} = (\boldsymbol{\omega}^{(12)} \cdot \mathbf{e}_f) - \kappa_h(\mathbf{v}^{(12)} \cdot \mathbf{e}_h) \tag{A5}$$

$$\begin{aligned} t_{33} = & -\mathbf{n} \cdot [(\boldsymbol{\omega}^{(1)} \times \mathbf{v}_{tr}^{(2)}) - (\boldsymbol{\omega}^{(2)} \times \mathbf{v}_{tr}^{(1)})] \\ & +(\mathbf{n} \times \boldsymbol{\omega}^{(12)}) \cdot \mathbf{v}^{(12)} - \kappa_f(\mathbf{v}^{(12)} \cdot \mathbf{e}_f)^2 - \kappa_h(\mathbf{v}^{(12)} \cdot \mathbf{e}_h)^2 \end{aligned} \tag{A6}$$

The nomenclature for equations (A4) to (A6) is described as follows:

$\boldsymbol{\omega}^{(1)}$ angular velocity of the generating tool

$\boldsymbol{\omega}^{(2)}$ angular velocity of the generated gear

$\boldsymbol{\omega}^{(12)}$ defined as $\boldsymbol{\omega}^{(1)}$ - $\boldsymbol{\omega}^{(2)}$

$\mathbf{v}_{tr}^{(1)}$ transfer motion velocity of the generating tool

$\mathbf{v}_{tr}^{(2)}$ transfer motion velocity of the generated gear

$\mathbf{v}^{(12)}$ defined as $\mathbf{v}_{tr}^{(1)} - \mathbf{v}_{tr}^{(2)}$

$\mathbf{n}$ surface unit normal vector

The equations discussed above are used in the TCA program for determination of the contact ellipse at the points of contact path of the modified helical gear drive.

Numerical Example

The input and output for the determination of the principal curvatures of the pinion tooth surface are shown in Tables A1 and A2. The input and output for the determination of the principal curvatures of the gear tooth surface are shown in Tables A3 and A4.

Table A1 Input Data

Description	Symbol	Values
first principal curvature of tool	κ_f	-0.0016 (1/mm)
second principal curvature of tool	κ_h	0.0 (1/mm)
first principal direction of tool	$\mathbf{e}_f$	$[0.0016 \ -0.4999 \ 0.8660]^T$
second principal direction of tool	$\mathbf{e}_h$	$[0.9387 \ 0.2992 \ 0.1710]^T$
angular velocity of tool (1/sec)	$\omega^{(1)}$	$[0.0 \ 0.0 \ 0.0]^T$
angular velocity of pinion (1/sec)	$\omega^{(2)}$	$[0.0 \ 0.0 \ 1.0]^T$
transfer velocity of tool (mm/sec)	$\mathbf{v}_{tr}^{(1)}$	$[0.0 \ 58.6588 \ 0.0]^T$
transfer velocity of pinion (mm/sec)	$\mathbf{v}_{tr}^{(2)}$	$[-0.1514 \ 58.5951 \ 0.0]^T$
surface normal of tangent point	$\mathbf{n}$	$[-0.3420 \ 0.8138 \ 0.4698]^T$

Table A2 Output Data

Description (for pinion)	Symbol	Values
first principal curvature	κ_s	-0.001543 (1/mm)
second principal curvature	κ_q	0.03907 (1/mm)
first principal direction	$\mathbf{e}_s$	$[0.1751 \ -0.4361 \ 0.8827]^T$
second principal direction	$\mathbf{e}_q$	$[0.9222 \ 0.3865 \ 0.0080]^T$

Table A3 Input Data

Description	Symbol	Values
first principal curvature of tool	κ_f	0.0 (1/mm)
second principal curvature of tool	κ_h	0.0 (1/mm)
first principal direction of tool	$\mathbf{e}_f$	$[0.9397\ 0.2962\ 0.1710]^T$
second principal direction of tool	$\mathbf{e}_h$	$[0.0\ -0.5\ 0.8660]^T$
angular velocity of tool (1/sec)	$\omega^{(1)}$	$[0.0\ 0.0\ 0.0]^T$
angular velocity of gear (1/sec)	$\omega^{(2)}$	$[0.0\ 0.0\ -0.2003]^T$
transfer velocity of tool (mm/sec)	$\mathbf{v}_{tr}^{(1)}$	$[0.0\ 58.6588\ 0.0]^T$
transfer velocity of gear (mm/sec)	$\mathbf{v}_{tr}^{(2)}$	$[-0.2096\ 58.7218\ 0.0]^T$
surface normal of tangent point	$\mathbf{n}$	$[-0.3420\ 0.8138\ 0.4698]^T$

Table A4 Output Data

Description (for gear)	Symbol	Values
first principal curvature	κ_s	-0.007836 (1/mm)
second principal curvature	κ_q	0.0 (1/mm)
first principal direction	$\mathbf{e}_s$	$[0.9219\ 0.3874\ 0.0]^T$
second principal direction	$\mathbf{e}_q$	$[0.1820\ -0.4331\ 0.8827]^T$

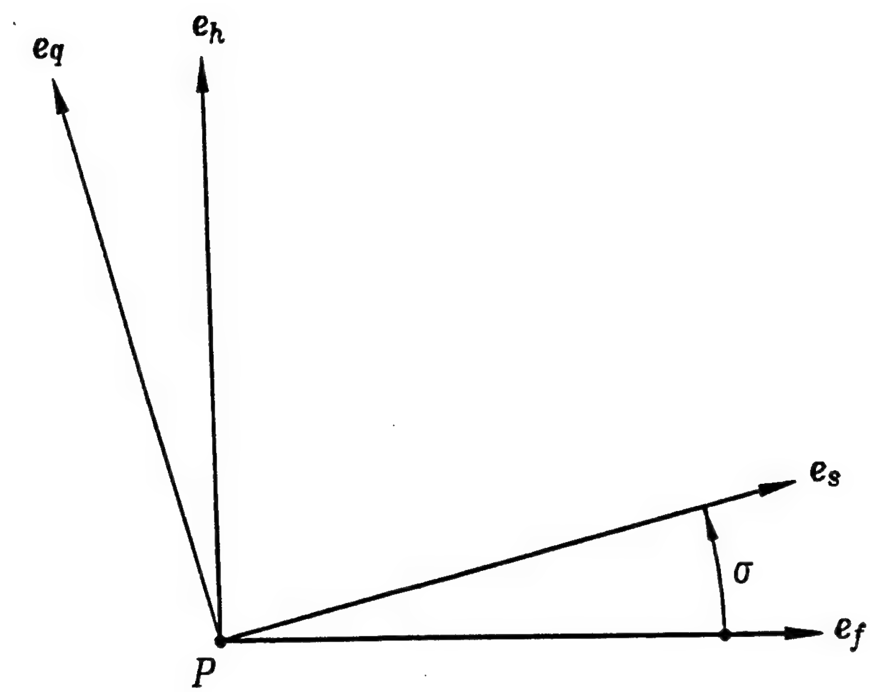


Fig. A1 Principal Directions

Appendix B. Contact Ellipse

Determination of Dimensions and Orientation of Instantaneous Contact Ellipse

The gear tooth surfaces are in point contact at every instant. Due to elastic deformation of gear tooth surfaces the contact is spread over an elliptical area and the center of the ellipse coincides with the instantaneous contact point. The bearing contact is formed as the set of instantaneous contact ellipses.

The dimensions and orientation of the instantaneous contact ellipse can be determined using the data about the principal curvatures and directions of the contacting surfaces, and the elastic approach of the surfaces. The elastic approach depends on the applied load but we will consider it as a given value that is known from experimental data.

The determination of the instantaneous contact ellipse is based on the following equations (proposed by Litvin):

$$\cos 2\alpha^{(1)} = \frac{g_1 - g_2 \cos 2\sigma}{(g_1^2 - 2g_1 g_2 \cos 2\sigma + g_2^2)^{1/2}} \tag{B1}$$

$$\sin 2\alpha^{(1)} = \frac{g_2 \sin 2\sigma}{(g_1^2 - 2g_1 g_2 \cos 2\sigma + g_2^2)^{1/2}} \tag{B2}$$

$$2a = 2\left|\frac{\delta}{A}\right|^{1/2}, \qquad 2b = 2\left|\frac{\delta}{B}\right|^{1/2} \tag{B3}$$

where

$$A = \frac{1}{4}[\kappa_\Sigma^{(1)} - \kappa_\Sigma^{(2)} - (g_1^2 - 2g_1 g_2 \cos 2\sigma + g_2^2)^{1/2}] \tag{B4}$$

$$B = \frac{1}{4}[\kappa_\Sigma^{(1)} - \kappa_\Sigma^{(2)} + (g_1^2 - 2g_1 g_2 \cos 2\sigma + g_2^2)^{1/2}] \tag{B5}$$

$$\kappa_\Sigma^{(i)} = \kappa_I^{(i)} + \kappa_{II}^{(i)}, \qquad g_i = \kappa_I^{(i)} - \kappa_{II}^{(i)} \tag{B6}$$

Here (fig. B1) $\alpha^{(1)}$ is the angle that is formed by axis η of the contact ellipse with the unit vector $\mathbf{e}_I^{(1)}$ of the principal direction on surface Σ_1; σ is the angle formed by unit vectors $\mathbf{e}_I^{(1)}$ and $\mathbf{e}_I^{(2)}$ of the principal directions of the contacting surfaces; $2a$ and $2b$ are the axes of the contact ellipse; δ is the elastic approach; and $\kappa_I^{(i)}$ and $\kappa_{II}^{(i)}$ are two principal directions of tooth surface i.

Numerical Example

The input and output for the determination of the contact ellipse are shown in Tables B1 and B2.

Table B1 Input Data

Description	Symbol	Values
pinion first principal curvature	κ_s	-0.001543 (1/mm)
pinion second principal curvature	κ_q	0.03907 (1/mm)
pinion first principal direction	$\mathbf{e}_s$	$[0.1751\ \ -0.4361\ 0.8827]^T$
pinion second principal direction	$\mathbf{e}_q$	$[0.9222\ 0.3865\ 0.0080]^T$
gear first principal curvature	κ_f	-0.007836 (1/mm)
gear second principal curvature	κ_h	0.0 (1/mm)
gear first principal direction	$\mathbf{e}_f$	$[-0.9219\ \ -0.3874\ 0.0]^T$
gear second principal direction	$\mathbf{e}_h$	$[-0.1820\ 0.4331\ 0.8827]^T$
elastic approach	δ	0.007 (mm)

Table B2 Output Data

Description	Symbol	Values
long axis of contact ellipse	$2a$	6.026 (mm)
short axis of contact ellipse	$2b$	1.092 (mm)
angle between long axis and the first principal direction of pinion	$\alpha^{(1)}$	89.87 (deg)

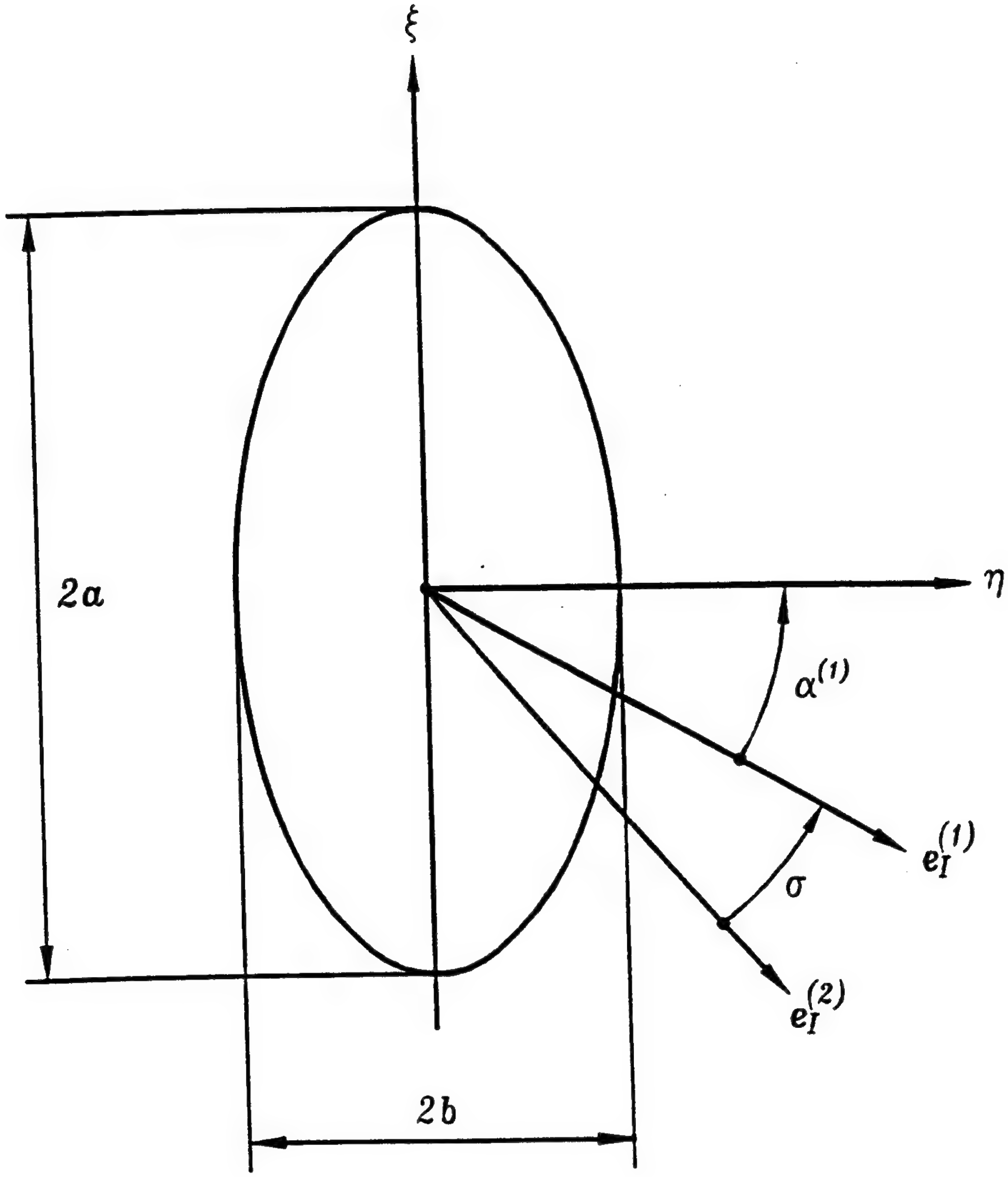


Fig. B1 Orientation and dimensions of contact ellipse

Appendix C. Directions for Users of Application of Computer Program

C.1 Introduction

The name of the computer program is HELTCA.FOR. It is written in FORTRAN77 language. The operating system is CMS-9.0. A subroutine DNEQNF to solve a system of nonlinear equations should be available in the Math-Library or working environment. The subroutine is not included in the program. The program will call the subroutine DNEQNF several times.

C.2 Input Block

The input block consists of three parts: (1) design parameters of pinion and gear; (2) the controlled modification parameters; and (3) parameters for TCA.

Part 1. Design parameters of pinion and gear

In the beginning of the computer program, you can read the following lines:

```
C... A11.....COEFFICIENT FOR TRANSFORMATION OF DEGREE TO RADIAN
     A11=DACOS(-1.D0)/180.D0
C
C... KHD=1 FOR RIGHT-HAND PINION AND LEFT-HAND GEAR
C... KHD=2 FOR LEFT-HAND PINION AND RIGHT-HAND GEAR

            KHD=2
```

If you write "KHD=1", the computer will use the necessary equations for the case of right-hand pinion and left-hand gear. The computations will be accomplished for a left-hand pinion and right-hand gear if you use "KHD=2".

Then, the variable definition for the pinion follows:

```
C
C... INPUT THE DESIGN PARAMETERS OF PINION
C...
C... TN1............GEAR NUMBER OF TEETH
C... PN1............NORMAL DIAMETRAL PITCH (1/MM)
C... PSIN1.........NORMAL PRESSURE ANGLE (RAD.)
C... BETAP1.....HELICAL ANGLE ON PITCH CYLINDER (RAD.)
C... ADG1.........ADDENDUM (MM)
```

```
C... DEG1..........DEDENDUM (MM)
C... LAMDP1.....LEAD ANGLE ON PITCH CYLINDER (RAD.)
C... FW1............FACE WIDTH (MM)
C... RPT1..........RADIUS OF PITCH CYLINDER (MM)
C... RBT1..........RADIUS OF BASE CYLINDER (MM)
C... RAT1..........RADIUS OF ADDENDUM CYLINDER (MM)
C... RDT1..........RADIUS OF DEDENDUM CYLINDER (MM)
C... PSIT1.........TRANSVERSE PRESSURE ANGLE (RAD.)
C... LAMDB1....LEAD ANGLE ON BASE CYLINDER (RAD.)
C
```

In accordance with our numerical example (see Table 1), the following data would be used:

```
TN1=20.D0
PN1=5.D0/25.4D0
PSIN1=A11*20.D0
LAMDP1=A11*60.D0
BETAP1=A11*30.D0
FW1=25.4D0*1.6D0
ADG1=1.D0/PN1
DEG1=1.25D0/PN1
```

The following variables are used for the gear:

```
C
C... INPUT THE DESIGN PARAMETERS OF GEAR
C
C... TN2..............GEAR NUMBER OF TEETH
C... PN2..............NORMAL DIAMETRAL PITCH (1/MM)
C... PSIN2..........NORMAL PRESSURE ANGLE (RAD.)
C... BETAP2.....HELICAL ANGLE ON PITCH CYLINDER (RAD.)
C... ADG2...........ADDENDUM (MM)
C... DEG2...........DEDENDUM (MM)
C... LAMDP2.....LEAD ANGLE ON PITCH CYLINDER (RAD.)
C... FW2..............FACE WIDTH (MM)
C... RPT2...........RADIUS OF PITCH CYLINDER (MM)
C... RBT2...........RADIUS OF BASE CYLINDER (MM)
C... RAT2...........RADIUS OF ADDENDUM CYLINDER (MM)
C... RDT2...........RADIUS OF DEDENDUM CYLINDER (MM)
C... PSIT2..........TRANSVERSE PRESSURE ANGLE (RAD.)
C... LAMDB2.....LEAD ANGLE ON BASE CYLINDER (RAD.)
C
```

From the design parameters listed in Table 1, we have

```
TN2=100.D0
PN2=5.D0/25.4D0
A11=DACOS(-1.D0)/180.D0
PSIN2=A11*20.D0
LAMDP2=A11*60.D0
BETAP2=A11*30.D0
FW2=25.4D0*1.6D0
ADG2=1.D0/PN2
DEG2=1.25D0/PN2
```

If KHD=2, the computer program will change the values of some design parameters as follows

```
C
      IF(KHD.EQ.2) THEN
         LAMDP1=-LAMDP1
         BETAP1=-BETAP1
         LAMDP2=-LAMDP2
         BETAP2=-BETAP2
      ENDIF
```

The computer program call the following subroutines "DATAT1" and "DATAT2" to calculate other tooth element proportions and output the whole data in file 55 (see below).

```
CALL DATAT1
CALL DATAT2
```

Part 2. Control of modification parameter for application in the TCA program

At this stage we can read

```
C
C..... THE FOLLOWING DATA IS FOR THE TO-BE CONTROLLED MODIFICATION
PARAMETERS
C
C... AA... MODIFICATION PARAMETER OF GEAR
C... AP... MODIFICATION PARAMETER OF PINION RACK-CUTTER
C... DP... TANGENT POINT N OF PROFILES OF PINION & GEAR RACK-CUTTERS
C... THET2P..INITIAL ANGLE FOR MODIFICATION OF GEAR (RAD.)
```

You must input the four parameters for modification of pinion and gear surfaces, for example:

```
AA=-0.0014D0
THET2P=-0.08D0
AP=-0.0008
DP=-DSIN(PSIN1)*DCOS(-1.D0)*RPT1/TN1/8.D0
```

The above four controlling parameters should be tried several times is order to obtain better contact pattern and transmission errors optimal for a given design.

Part 3. Parameters for TCA

In this part, the alignment errors expected should be input:

```
C
C    PARAMETERS FOR TCA
C
C.. KM .. SWITCH 1 FOR CROSSING ANGLE Δγx & 2 FOR INTERSECTION
ANGLE Δγy
C.. DGAM..ANGLE OF MISALIGNMENT(CROSSING OR INTERSECTION) (ARC
MINUTE)
C.. DEE... CHANGE OF CENTER DISTANCE (MM)
C
```

If a crossing angle of misalignment is considered, should input "KM=1". Input "KM=2" if an intersection angle of misalignment is considered. For an aligned gear drive, input "DGAM=0.0". For instance, if $\Delta\gamma_x = 4'$, $\Delta E = 0$, input the following lines:

```
        KM=1
C       KM=2
        DGAM=4.0*A11/60.D0
        DEE=0.000D0
```

Then you will read the following sentence:

```
C
C.... THE INPUT BLOCK IS READY
```

Usually, you cannot make changes anything after this step.

C.3 Output Block

After the input block is filled out, you can read the following explanation for the output files:

```
C
C... OUTPUT DATA FILES ARE THE FOLLOWINGS
C
C... FILE 55... TOOTH PROPORTIONS OF PINION AND GEAR
C... FILE 85... CONTACT PATH ON PINION SURFACE (2D)
C... FILE 86... CONTACT PATH ON GEAR SURFACE (2D)
C... FILE 87... LENGTH AND DIRECTIONS OF CONTACT ELLIPSE ON PINION
AND GEAR SURFACES (2D)
```

```
C... FILE 90... TRANSMISSION ERRORS
C
C...................
C
```

1) File 55

In File 55 the information about the pinion and gear are listed.

2) Files 85 and 86

There are two coordinates in File 85 for each contact point of the pinion: Radial $(x_p^2+y_p^2)^{0.5}$ and axial z_p . There are two coordinates in File 86 for each contact point of the gear: Radial $(x_g^2+y_g^2)^{0.5}$ and axial z_g.

3) File 87

There are 5 values in File 87 for each pair of contact points of the pinion and gear. The first one is the value of the major semi-axis of the contact ellipse. The second and third values are the cosine directions of the major axis of the contact ellipse on the pinion tooth surface. The last two values are the cosine direction of the major axis of the contact ellipse on the gear tooth surface.

C.4 Computer Program

```
C............... TCA FOR MODIFIED HELICAL GEARS..................
      PROGRAM HELTCA
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 XI(9),X(9),F(9)
      REAL*8 LF1(3,3),LH2(3,3),L1F(3,3),L2H(3,3)
      REAL*8 R1F(3),R2F(3),N1F(3),N2F(3)
      REAL*8 DPHI(2,180),DDPHI(2,180,4)
      REAL*8 ELAL(180),EL1(2,180),EL2(2,180)
      REAL*8 LFH(3,3),LHF(3,3)
      REAL*8 V1(3),V2(3),V3(3),V4(3),V5(3)
      REAL*8 RG2(3),NG2(3)
      REAL*8 RG1(3),NG1(3)
      REAL*8 UI(3),UJ(3),UK(3)
      REAL*8 LAMDP2,LAMDB2
      REAL*8 LAMDP1,LAMDB1
      REAL*8 UT2,KT21,KT22
      REAL*8 AVC1(3),VTR1(3),AVC2(3),VTR2(3)
      REAL*8 KSIG1,KSIG2,KFF,KHH
      REAL*8 EFN(3),EHN(3),W1VT2(3),WV12(3),W2VT1(3),KF,KH,KS,KQ
      REAL*8 KM2,KT2
      REAL*8 KFP,KHP,KFG,KHG
      COMMON /A300/ ES(3),EQ(3)
      COMMON /A310/ KFF,KHH
      COMMON /A340/ EFF(3),EHH(3)
      COMMON /A360/ A,B,SI(3),FI(3)
      COMMON/A200/ W1(3),W2(3),W12(3),VT1(3),VT2(3),V12(3)
      COMMON /A210/ EX(3),EF(3),EH(3)
      COMMON /A212/ EF2(3),EH2(3),KF2,KH2
      COMMON /A220/ KF,KH
      COMMON /A230/ ET(3),EM(3)
      COMMON /A380/ KS,KQ
      COMMON /A400/ VT11(3),VT12(3),VT21(3),VT22(3)
      COMMON /A401/ KHP,KFP,KHG,KFG
      EXTERNAL FCNG,FCNC,FCNT
      COMMON /AXIS/ UI,UJ,UK
      COMMON /NET/ RR,DD
      COMMON /DD/ DF,KPRI
      COMMON /DATT2/ TN2,PN2,PSIN2,BETAP2,ADG2,DEG2,LAMDP2,
     & UP2,FW2,RPT2,RBT2,RAT2,RDT2,PSIT2,LAMDB2
      COMMON /DATT/ TN1,PN1,PSIN1,BETAP1,ADG1,DEG1,LAMDP1,
     & FW1,RPT1,RBT1,RAT1,RDT1,PSIT1,LAMDB1
      COMMON /B2/XNP1,YNP1,ZNP1,XNP2,YNP2,ZNP2
      COMMON /B4/X1,Y1,Z1,XN1,YN1,ZN1
      COMMON /B5/X2,Y2,Z2,XN2,YN2,ZN2
      COMMON /B6/THET2P,DGPHI2
      COMMON /W1/ ETAW1,UPP1,SPP1,ETAW2,UPP2,SPP2
      COMMON /SG1/ RG1,NG1,AP,DP
      COMMON /SG2/ RG2,NG2,AA
      COMMON /ATT/ PHI1,PHI2,R1F,R2F,N1F,N2F,LFH,CC,DGAM,DPHI2
      COMMON /MVT/ LF1,LH2
      COMMON /AST/ ICONT
      COMMON /ATS/ DPHI1
C... A11.....COEFFICIENT FOR TRANSFORMATION OF DEGREE TO RADIAN
      A11=DACOS(-1.D0)/180.D0
C... KHD=1 FOR RIGHT-HAND PINION AND LEFT-HAND GEAR
C... KHD=2 FOR LEFT-HAND PINION AND RIGHT-HAND GEAR
      KHD=2
C... INPUT THE DESIGN PARAMETERS OF PINION
C... TN1........GEAR NUMBER OF TEETH
C... PN1........NORMAL DIAMETRAL PITCH (MM)
```

```
C... PSIN1......NORMAL PRESSURE ANGLE (RAD.)
C... BETAP1.....LEADING ANGLE OF THE HELIX ON PITCH CYLINDER (RAD.)
C... ADG1....... ADDENDUM (MM)
C... DEG1....... DEDENDUM (MM)
C... LAMDP1....HELIX ANGLE ON PITCH CYLINDER (RAD.)
C... FW1........FACE WIDTH  (MM)
C... RPT1.......RADIUS OF PITCH CYLINDER (MM)
C... RBT1.......RADIUS OF BASE CYLINDER  (MM)
C... RAT1.......RADIUS OF ADDENDUM CYLINDER  (MM)
C... RDT1.......RADIUS OF DEDENDUM CYLINDER  (MM)
C... PSIT1......TRANSVERSE PRESSURE ANGLE    (RAD.)
C... LAMDB1....HELIX ANGLE ON BASE CYLINDER  (RAD.)
      TN1=20.D0
      PN1=5.D0/25.4D0
      PSIN1=A11*20.D0
      LAMDP1=A11*60.D0
      BETAP1=A11*30.D0
      FW1=25.4D0*1.6D0
      ADG1=1.D0/PN1
      DEG1=1.25D0/PN1
C... INPUT THE DESIGN PARAMETERS OF GEAR
C... TN2........GEAR NUMBER OF TEETH
C... PN2.........NORMAL DIAMETRAL PITCH (1/MM)
C... PSIN2.......NORMAL PRESSURE ANGLE  (RAD.)
C... BETAP2......LEADING ANGLE OF THE HELIX ON PITCH CYLINDER (RAD.)
C... ADG2........ ADDENDUM  (MM)
C... DEG2........ DEDENDUM  (MM)
C... LAMDP2.....HELIX ANGLE ON PITCH CYLINDER  (RAD.)
C... FW2.........FACE WIDTH   (MM)
C... RPT2........RADIUS OF PITCH CYLINDER  (MM)
C... RBT2........RADIUS OF BASE CYLINDER   (MM)
C... RAT2........RADIUS OF ADDENDUM CYLINDER  (MM)
C... RDT2........RADIUS OF DEDENDUM CYLINDER  (MM)
C... PSIT2.......TRANSVERSE PRESSURE ANGLE    (RAD.)
C... LAMDB2.....HELIX ANGLE ON BASE CYLINDER  (RAD.)
      TN2=100.D0
      PN2=5.D0/25.4D0
      A11=DACOS(-1.D0)/180.D0
      PSIN2=A11*20.D0
      LAMDP2=A11*60.D0
      BETAP2=A11*30.D0
      FW2=25.4D0*1.6D0
      ADG2=1.D0/PN2
      DEG2=1.25D0/PN2
      IF(KHD.EQ.2) THEN
        LAMDP1=-LAMDP1
        BETAP1=-BETAP1
        LAMDP2=-LAMDP2
        BETAP2=-BETAP2
      ENDIF
      CALL DATAT1
      CALL DATAT2
C... THE FOLLOWING IS FOR CONTROLLING PARAMETERS
C... AA... MODIFICATION PARAMETER OF GEAR
C... AP... MODIFICATION PARAMETER OF PINION  RACK-CUTTER
C..  DP....TANGENT POINT N OF PROFILES OF PINION & GEAR RACK-CUTTERS
C..  THET2P..INITIAL ANGLE FOR MODIFICATION OF GEAR (RAD.)
C
C    PARAMETERS FOR TCA
C
```

```
C..  KM .. SWITCH 1 FOR CROSSING & 2 FOR INTERSECTING MISALIGNMENT
C..  DGAM..ANGLE OF DISALIGNMENT(CSOSSING OR INTERSECTING)(ARC MINUTE)
C..  DEE... CHANGE OF CENTER DISTANCE  (MM)
      KM=1
C     KM=2
      DGAM=0.D0
      DEE=0.000D0
      AA=-0.0014D0
      THET2P=-0.08D0
      AP=-0.0008
      DP=-DSIN(PSIN1)*DCOS(-1.D0)*RPT1/TN1/8.D0
C... THE INPUT BLOCK IS OVER HERE
C
C... OUPTPUT DATA FILES ARE THE FOLLOWINGS
C
C... FILE 55... TOOTH PROPORTIONS OF PINION AND GEAR
C... FILE 85... CONTACT PATH ON PINION SURFACE  (2D)
C... FILE 86... CONTACT PATH ON GEAR SURFACE  (2D)
C... FILE 87... DIRECTIONS OF LONG AXIS OF CONTACT ELLIPS (2D)
C... FILE 90 ...TRANSMISSION ERRORS
      DO 901 I=1,3
        UI(I)=0.D0
        UJ(I)=0.D0
        UK(I)=0.D0
  901 CONTINUE
      UI(1)=1.D0
      UJ(2)=1.D0
      UK(3)=1.D0
C .. EE2..... GEAR RATIO
      EE2=TN1/TN2
C .. CC...... CENTER DISTANCE OF GEAR DRIVE
      CC=RPT1+RPT2+DEE
C .. CALCULATE CONTACT POINT ON MEAN SECTION WITHOUT MISALIGNMENT
      ICONT=1
      N=6
      ERRREL=0.1D-6
      ITMAX=1000
C
      CALL MIAL(KM,0.D0,LFH)
C
      XI(1)=0.D0
      XI(2)=0.D0
      XI(3)=0.D0
      XI(4)=0.D0
      XI(5)=0.D0
      XI(6)=0.D0
      DD=0.D0
C
      CALL DNEQNF(FCNG,ERRREL,N,ITMAX,XI,X,FNORM)
C
      PHISS1=X(6)
      PHI1MEA=X(6)
C .. CALCULATE CONTACT POINT ON EDGE SECTION WITHOUT MISALIGNMENT
      DD=-0.5D0*FW1
      ERRREL=0.1D-8
C
      CALL DNEQNF(FCNG,ERRREL,N,ITMAX,XI,X,FNORM)
C
      PHISS2=X(6)
C .. CALCULATE CONTACT POINT ON EDGE SECTION  WITH MISALIGNMENT
```

```
      N=6
      ERRREL=0.1D-6
      ITMAX=1000
C
      CALL MIAL(KM,DGAM,LFH)
C
      XI(1)=X(1)
      XI(2)=X(2)
      XI(3)=X(3)
      XI(4)=X(4)
      XI(5)=X(5)
      XI(6)=X(6)
C
      CALL DNEQNF(FCNG,ERRREL,N,ITMAX,XI,X,FNORM)
C
 1330 CONTINUE
      PHI1=X(6)
      PHI1STA=X(6)
C .. THE FOLLOWING IS FOR TCA
      ICONT=2
      N=5
      ERRREL=0.1D-5
      ITMAX=400
      STP=(PHISS2-PHISS1)/36.D0
      NN=72
      DO 1010 I=1,NN
        PHI1=PHI1STA-(I-1)*STP
        XI(1)=X(1)
        XI(2)=X(2)
        XI(3)=X(3)
        XI(4)=X(4)
        XI(5)=X(5)
C
        CALL DNEQNF(FCNG,ERRREL,N,ITMAX,XI,X,FNORM)

        IF(RG1(3).GT.(0.5D0*FW1)) GO TO 1011
C...  ROTATING VELOCITY OF CUTTER IN CUTTER SYSTEM
        W1(1)=0.D0
        W1(2)=0.D0
        W1(3)=0.D0
C... ROTATING VELOCITY OF PINION IN CUTTER SYSTEM
        W2(1)=0.D0
        W2(2)=0.D0
        W2(3)=1.D0
C... NORMAL OF CUUTER IN CUTTER SYSTEM
        EX(1)=XNP1
        EX(2)=YNP1
        EX(3)=ZNP1
C... TRANSFER VELOCITIES OF CUTTER AND PINION IN CUTTER SYSTEM
        CALL EQVEC(VT1,VT11)
        CALL EQVEC(VT2,VT12)
C... RELATIVE VELOCITIS OF CUTTER WRT. PINION IN CUTTER SYSTEM
        CALL ADDVEC(W12,W1,W2,-1.D0)
        CALL ADDVEC(V12,VT1,VT2,-1.D0)
        KF=KFP
        KH=KHP
C... PRINCIPAL CURVATUTES AND DIRECTIONS OF PINION
        CALL CURVT(1)
C... PRINCIPAL DIRECTIONS OF PINION IN PINION SYSTEM
        CALL EQVEC(EFF,ES)
```

```
      CALL EQVEC(EHH,EQ)
C... ROTATING VELOCITY OF CUTTER IN CUTTER SYSTEM
      W1(1)=0.D0
      W1(2)=0.D0
      W1(3)=0.D0
C... ROTATING VELOCITY OF GEAR IN CUTTER SYSTEM
      W2(1)=0.D0
      W2(2)=0.D0
      W2(3)=-DGPHI2
C... NORMAL OF CUUTER IN CUTTER SYSTEM
      EX(1)=XNP2
      EX(2)=YNP2
      EX(3)=ZNP2
C... TRANSFER VELOCITIES OF CUTTER AND PINION
      CALL EQVEC(VT1,VT21)
      CALL EQVEC(VT2,VT22)
      CALL EQVEC(EF,EF2)
      CALL EQVEC(EH,EH2)
      KF=KF2
      KH=KH2
C... RELATIVE VELOCITIS OF CUTTER WRT. GEAR IN CUTTER SYSTEM
      CALL ADDVEC(W12,W1,W2,-1.D0)
      CALL ADDVEC(V12,VT1,VT2,-1.D0)
      KF=KFG
      KH=KHG
C... PRINCIPAL CURVATUTES AND DIRECTIONS OF PINION
      CALL CURVT(2)
C... PRINCIPAL DIRECTIONS OF PINION IN PINION SYSTEM
      CALL TRANSM(L1F,LF1)
      CALL MAVEC(V1,LH2,ES)
      CALL MAVEC(V2,LFH,V1)
      CALL MAVEC(ES,L1F,V2)
      CALL MAVEC(V1,LH2,EQ)
      CALL MAVEC(V2,LFH,V1)
      CALL MAVEC(EQ,L1F,V2)
C... CONTACT ELLIPSES
      CALL ELLIP
      ELAL(I)=A
C... AXIS OF PINION ELLIPSE ON TANGENT PLANE
      CALL EQVEC( V1,SI)
      CALL DOTVEC(V1N,V1,V1)
      EL1(1,I)=DSQRT(V1(1)**2+V1(2)**2/V1N)
      EL1(2,I)=V1(3)/DSQRT(V1N)
C... AXIS OF GEAR ELLIPSE ON TANGENT PLANE
      CALL TRANSM(LHF,LFH)
      CALL TRANSM(L2H,LH2)
      CALL MAVEC(V2,LF1,V1)
      CALL MAVEC(V1,LHF,V2)
      CALL MAVEC(V2,L2H,V1)
      CALL EQVEC(V1,V2)
      CALL DOTVEC(V1N,V1,V1)
      EL2(1,I)=-DSQRT(V1(1)**2+V1(2)**2/V1N)
      EL2(2,I)=V1(3)/DSQRT(V1N)
      AG1=DATAN(EL1(2,I)/EL1(1,I))/A11
      AG2=DATAN(EL2(2,I)/EL2(1,I))/A11
      DPHI(1,I)=PHI1
      DPHI(2,I)=PHI2-TN1/TN2*PHI1
      DDR1=DSQRT(RG1(1)**2+RG1(2)**2)
      DDR2=DSQRT(RG2(1)**2+RG2(2)**2)-210.D0
      KM=4
```

```
      OU1=FLOAT(I-1)/FLOAT(KM)
      OUP=AINT(OU1)
      IF(OU1.EQ.OUP) THEN
        WRITE(85,*) DDR1,RG1(3)
        WRITE(86,*) DDR2,RG2(3)
C       WRITE(87,*) ELAL(I),EL1(1,I),EL1(2,I),EL2(1,I),EL2(2,I)
      ENDIF
 1010 CONTINUE
 1011 CONTINUE
      AD=-10000.D0
      DO 1090 I=1,NN,KM
        IF(DPHI(2,I).NE.0.D0) THEN
          IF(DPHI(2,I).GT.AD) THEN
            AD=DPHI(2,I)
          ENDIF
        ENDIF
 1090 CONTINUE
      SS=360.D0/TN1
      DO 1020 J=1,4
        DO 1030 I=1,NN,KM
          BD=(DPHI(2,I)-AD)/A11*3600.D0
          DDPHI(1,I,J)=DPHI(1,I)/A11+SS*(J-1)
          DDPHI(2,I,J)=BD
 1030   CONTINUE
 1020 CONTINUE
      KM=4
      DO 1040 J=1,4
        DO 1050 I=1,NN,KM
          IF(DDPHI(1,I,J).NE.0.D0) THEN
            WRITE(90,*) DDPHI(1,I,J),DDPHI(2,I,J)
          ENDIF
 1050   CONTINUE
 1040 CONTINUE
      KM=4
      DO 1060 I=1,NN,KM
        WRITE(87,*) ELAL(I),EL1(1,I),EL1(2,I),EL2(1,I),EL2(2,I)
 1060 CONTINUE
      WRITE(6,*) '***** PROGRAM FINISHED ******'
      STOP
      END
C
C ...THE SUBROUTINE  IS FOR TCA
C...
      SUBROUTINE FCNG(X,F,N)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 X(N),F(N)
      REAL*8 LF1(3,3),LH2(3,3),LFH(3,3)
      REAL*8 R1F(3),R2F(3),N1F(3),N2F(3),CO(3)
      REAL*8 AVC1(3),VTR1(3),AVC2(3),VTR2(3)
      REAL*8 UI(3),UJ(3),UK(3),V1(3),V2(3),V3(3),V4(3),V5(3)
      REAL*8 UP,LAMDP,LAMDB
      REAL*8 KF2,KH2
      REAL*8 KFP,KHP,KFG,KHG
      COMMON /AXIS/ UI,UJ,UK
      COMMON /SG1/ RG1,NG1,AP,DP
      COMMON /SG2/ RG2,NG2,AA
      COMMON /ATT/ PHI1,PHI2,R1F,R2F,N1F,N2F,LFH,CC,DGAM,DPHI2
      COMMON /AST/ ICONT
      COMMON /MVT/ LF1,LH2
      REAL*8 KSIG1,KSIG2,KFF,KHH
```

```
      REAL*8 EFN(3),EHN(3),W1VT2(3),WV12(3),W2VT1(3),KF,KH,KS,KQ
      REAL*8 KM2,KT2
      COMMON /A300/ ES(3),EQ(3)
      COMMON /A310/ KFF,KHH
      COMMON /A340/ EFF(3),EHH(3)
      COMMON /A340/ EF1(3),EH1(3)
      COMMON /A360/ A,B,SI(3),FI(3)
      COMMON/A200/ W1(3),W2(3),W12(3),VT1(3),VT2(3),V12(3)
      COMMON /A210/ EX(3),EF(3),EH(3)
      COMMON /A212/ EF2(3),EH2(3),KF2,KH2
      COMMON /A220/ KF,KH
      COMMON /A230/ ET(3),EM(3)
      COMMON /A380/ KS,KQ
      COMMON /A400/ VT11(3),VT12(3),VT21(3),VT22(3)
      COMMON /A401/ KHP,KFP,KHG,KFG
      REAL*8 RG1(3),NG1(3)
      REAL*8 RG2(3),NG2(3)
      REAL*8 LAMDP2,LAMDB2
      REAL*8 LAMDP1,LAMDB1
      COMMON /DATT2/ TN2,PN2,PSIN2,BETAP2,ADG2,DEG2,LAMDP2,
     & UP2,FW2,RPT2,RBT2,RAT2,RDT2,PSIT2,LAMDB2
      COMMON /B2/XNP1,YNP1,ZNP1,XNP2,YNP2,ZNP2
      COMMON /B4/X1,Y1,Z1,XN1,YN1,ZN1
      COMMON /B5/X2,Y2,Z2,XN2,YN2,ZN2
      COMMON /B6/THET2P,DGPHI2
      COMMON /W1/ ETAW1,UPP1,SPP1,ETAW2,UPP2,SPP2
      COMMON /NET/ RR,DD
      COMMON /DATT/ TN1,PN1,PSIN1,BETAP1,ADG1,DEG1,LAMDP1,
     & FW1,RPT1,RBT1,RAT1,RDT1,PSIT1,LAMDB1
      UPP1=X(1)
      ETAW1=X(2)
      UPP2=X(3)
      ETAW2=X(4)
      PHI2=X(5)
      IF(ICONT.EQ.1) THEN
        PHI1=X(6)
      ENDIF
      RPT=RPT1
      CNST=DARCOS(-1.0D00)/180.0
      PI=DARCOS(-1.0D00)
      PI2=2.D0*PI
      PSIT= PSIT1
      PSIN= PSIN1
      BETAP= BETAP1
      DSIN1=DSIN(ETAW1)
      DCOS1=DCOS(ETAW1)
      ANF1=DARCOS(RBT1/RPT1)
      CINV1=DTAN(ANF1)-ANF1
      ANG1=PI/2.D0/TN1
      AM=PI/PN1/4.D0
      SA=AM*DCOS(BETAP)
      SS1= RPT1*(ETAW1)
      DSS1= RPT1
C... EQUATION OF MESHING OF PINION
      FF1=SPP1*DSIN(BETAP)
      FF2=(-(UPP1+DP)-2.D0*AP**2*UPP1**3)
      FF3=DSIN(PSIN)+2.D0*AP*UPP1*DCOS(PSIN)
      F5=FF2/FF3*DCOS(BETAP)-SA-RPT1*ETAW1
      SPP1= F5/DSIN(BETAP)
C... SURFACE OF PINION RACK CUTTER
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      XP=(UPP1+DP)*DCOS(PSIN)
      YP=((UPP1+DP)*DSIN(PSIN)+AM)*DCOS(BETAP)+
     & AP*UPP1**2*DCOS(PSIN)*DCOS(BETAP)+SPP1*DSIN(BETAP)
      ZP=-((UPP1+DP)*DSIN(PSIN)+AM)*DSIN(BETAP)-
     & AP*UPP1**2*DCOS(PSIN)*DSIN(BETAP)+SPP1*DCOS(BETAP)
      X1= DCOS(ETAW1)*XP+DSIN(ETAW1)*YP+RPT1*DCOS(ETAW1)
     &+SS1*DSIN(ETAW1)
      Y1=-DSIN(ETAW1)*XP+DCOS(ETAW1)*YP-RPT1*DSIN(ETAW1)
     &+SS1*DCOS(ETAW1)
      Z1=ZP
C... NORMAL OF RACK CUTTER
      PNN=DSQRT(1.D0+(2.D0*AP*UPP1)**2)
      XNPP=-2.D0*AP*UPP1/PNN
      YNPP=1.D0/PNN
      ZNPP=0.D0
      XNP1=(DCOS(PSIN)*XNPP-DSIN(PSIN)*YNPP)
      YNP1=(DSIN(PSIN)*DCOS(BETAP)*XNPP+DCOS(PSIN)*DCOS(BETAP)*YNPP)
      ZNP1=(-DSIN(PSIN)*DSIN(BETAP)*XNPP-DCOS(PSIN)*DSIN(BETAP)*YNPP)
C .. NORMAL OF PINION IN S1
      XN1= DCOS(ETAW1)*XNP1+DSIN(ETAW1)*YNP1
      YN1=-DSIN(ETAW1)*XNP1+DCOS(ETAW1)*YNP1
      ZN1= ZNP1
      DX1=-DSIN(ETAW1)*XP+DCOS(ETAW1)*YP+SS1*DCOS(ETAW1)
      DY1=-DCOS(ETAW1)*XP-DSIN(ETAW1)*YP-SS1*DSIN(ETAW1)
      DZ1=0.D0
      RG1(1)=X1
      RG1(2)=Y1
      RG1(3)=Z1
      NG1(1)=XN1
      NG1(2)=YN1
      NG1(3)=ZN1
      KHP=0.D0
      KFP=2.D0*AP/(DSQRT((1.D0+(2.D0*AP*UPP1)**2))**3)
      EF1(1)=DCOS(PSIN)
      EF1(2)=DSIN(PSIN)*DCOS(BETAP)
      EF1(3)=-DSIN(PSIN)*DSIN(BETAP)
      EH1(1)=0.D0
      EH1(2)= DSIN(BETAP)
      EH1(3)= DCOS(BETAP)
      EF(1)= DCOS(ETAW1)*EH1(1)+DSIN(ETAW1)*EH1(2)
      EF(2)=-DSIN(ETAW1)*EH1(1)+DCOS(ETAW1)*EH1(2)
      EF(3)= EH1(3)
      EH(1)= DCOS(ETAW1)*EF1(1)+DSIN(ETAW1)*EF1(2)
      EH(2)=-DSIN(ETAW1)*EF1(1)+DCOS(ETAW1)*EF1(2)
      EH(3)= EF1(3)
C     VELOCITY OF RACK-CUTTER IN SP
      VT11(1)=0.D0
      VT11(2)=RPT1
      VT11(3)=0.D0
C     VELOCITY OF PINION IN SP
      VT12(1)=-(YP+SS1)
      VT12(2)= XP+RPT1
      VT12(3)=0.D0
      RPT=RPT2
      PSIT= PSIT2
      PSIN= PSIN2
      BETAP= BETAP2
      SS2= RPT2*(ETAW2/TN2*TN1)
      DSS2= RPT2/TN2*TN1
      TTD=AA*(ETAW2-THET2P)**2
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      DTP=2.D0*AA*(ETAW2-THET2P)
      GPHI2= ETAW2/TN2*TN1-TTD
      DGPHI2= 1.D0/TN2*TN1-DTP
C... EQUATION OF MESHING OF GEAR
      GG1=SPP2*DSIN(PSIN2)*DSIN(BETAP2)
      GG1=DSIN(PSIN2)*DSIN(BETAP2)
      GG2=(UPP2+DP)*DCOS(BETAP2)
      GG3= RPT2*TN1/TN2*ETAW2*DSIN(PSIN2)
      GG4=SA*DSIN(PSIN2)
      GG5= 2.D0*AA*TN2*(ETAW2-THET2P)*RPT2*DCOS(PSIN2)*DCOS(BETAP2)
      GG6= TN1-2.D0*AA*TN2*(ETAW2-THET2P)
      SPP2=-(GG2+GG3+GG4+GG5/GG6)/GG1
C... SURFACE OF GEAR RACK CUTTER
      XP=(UPP2+DP)*DCOS(PSIN)
      YP=((UPP2+DP)*DSIN(PSIN)+AM)*DCOS(BETAP)+SPP2*DSIN(BETAP)
      ZP=-((UPP2+DP)*DSIN(PSIN)+AM)*DSIN(BETAP)+SPP2*DCOS(BETAP)
      X2=-DCOS(GPHI2)*XP+DSIN(GPHI2)*YP+RPT2*DCOS(GPHI2)
     &+SS2*DSIN(GPHI2)
      Y2=-DSIN(GPHI2)*XP-DCOS(GPHI2)*YP+RPT2*DSIN(GPHI2)
     &-SS2*DCOS(GPHI2)
      Z2=ZP
C... NORMAL OF RACK CUTTER IN SW
      XNP2=DSIN(PSIN)
      YNP2=-DCOS(PSIN)*DCOS(BETAP)
      ZNP2= DCOS(PSIN)*DSIN(BETAP)
      XNP2=-DSIN(PSIN)
      YNP2= DCOS(PSIN)*DCOS(BETAP)
      ZNP2=-DCOS(PSIN)*DSIN(BETAP)
C .. NORMAL OF GEAR IN S2
      XN2=-DCOS(GPHI2)*XNP2+DSIN(GPHI2)*YNP2
      YN2=-DSIN(GPHI2)*XNP2-DCOS(GPHI2)*YNP2
      ZN2= ZNP2
      DX2=(DSIN(GPHI2)*XP+DCOS(GPHI2)*YP-RPT2*DSIN(GPHI2)+
     &SS2*DCOS(GPHI2))*DGPHI2+DSS2*DSIN(GPHI2)
      DY2=(-DCOS(GPHI2)*XP+DSIN(GPHI2)*YP+RPT2*DCOS(GPHI2)+
     &SS2*DSIN(GPHI2))*DGPHI2-DSS2*DCOS(GPHI2)
      DZ2=0.D0
      RG2(1)=X2
      RG2(2)=Y2
      RG2(3)=Z2
      NG2(1)=XN2
      NG2(2)=YN2
      NG2(3)=ZN2
      KFG=0.D0
      KHG=0.D0
      EF2(1)=DCOS(PSIN)
      EF2(2)=DSIN(PSIN)*DCOS(BETAP)
      EF2(3)=-DSIN(PSIN)*DSIN(BETAP)
      EH2(1)=0.D0
      EH2(2)= DSIN(BETAP)
      EH2(3)= DCOS(BETAP)
C     VELOCITY OF RACK-CUTTER IN SG
      VT21(1)=0.D0
      VT21(2)= RPT1
      VT21(3)=0.D0
C     VELOCITY OF GEAR
      VT22(1)=-(YP+SS2)*DGPHI2
      VT22(2)= (XP+RPT2)*DGPHI2
      VT22(3)=0.D0
      S1=DSIN(PHI1)
```

```
      C1=DCOS(PHI1)
      S2=DSIN(PHI2)
      C2=DCOS(PHI2)
      LF1(1,1)=C1
      LF1(1,2)=S1
      LF1(2,1)=-S1
      LF1(2,2)=C1
      LF1(1,3)=0.D0
      LF1(2,3)=0.D0
      LF1(3,1)=0.D0
      LF1(3,2)=0.D0
      LF1(3,3)=1.D0
      LH2(1,1)=C2
      LH2(1,2)=-S2
      LH2(2,1)=S2
      LH2(2,2)=C2
      LH2(1,3)=0.D0
      LH2(2,3)=0.D0
      LH2(3,1)=0.D0
      LH2(3,2)=0.D0
      LH2(3,3)=1.D0
      CALL MAVEC(V1,LH2,NG2)
      CALL MAVEC(N2F,LFH,V1)
      CALL MAVEC(V2,LH2,RG2)
      CALL MAVEC(V3,LFH,V2)
      CALL ADDVEC(R2F,V3,UI,CC)
      CALL MAVEC(N1F,LF1,NG1)
      CALL MAVEC(R1F,LF1,RG1)
      F(1)=(R1F(1)-R2F(1))
      F(2)=(R1F(2)-R2F(2))
      F(3)=(R1F(3)-R2F(3))
      F(4)=N1F(1)-N2F(1)
      F(5)=N1F(2)-N2F(2)
      IF(ICONT.EQ.1) THEN
        F(6)=Z1-DD
      ENDIF
      RETURN
      END
C...
C... FOR PINION DATA **
C...
      SUBROUTINE DATAT1
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 UP,LAMDP1,LAMDB1
      COMMON /DATT/ TN1,PN1,PSIN1,BETAP1,ADG1,DEG1,LAMDP1,
     & FW1,RPT1,RBT1,RAT1,RDT1,PSIT1,LAMDB1
      RPT1=TN1/(2.D0*PN1*DCOS(BETAP1))
      PSIT1=DATAN(DTAN(PSIN1)/DCOS(BETAP1))
      RBT1=RPT1*DCOS(PSIT1)
      RAT1=RPT1+ADG1
      RDT1=RPT1-DEG1
      LAMDB1=DATAN(DTAN(LAMDP1)/DCOS(PSIT1))
      BETAB1=DATAN(DTAN(BETAP1)*DCOS(PSIT1))
      WRITE(55,110)
 110  FORMAT(/2X,'*******************************************',/
     &        2X,'*                 DATA OF PINION          *',/
     &        2X,'*******************************************',/)
      WRITE(55,120) TN1,PN1,PSIN1,BETAP1,ADG1,DEG1,LAMDP1,FW1,
     & RPT1,RBT1,RAT1,RDT1,PSIT1,LAMDB1
  120 FORMAT(2X,'GEAR NUMBER OF TEETH                 TN1=',F14.7/
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&2X,'NORMAL DIAMETRAL PITCH (1/MM)       PN1=',F14.7/
&2X,'NORMAL PRESSURE ANGLE  (RAD.)       PSIN1=',F14.7/
&2X,'LEADING ANGLE OF HELIX'              /
&4X,'ON PITCH CYLINDER        (RAD.)   BETAP1=',F14.7/
&2X,'ADDENDUM                 (MM)       ADG1=',F14.7/
&2X,'DEDENDUM                 (MM)       DEG1=',F14.7/
&2X,'HELIX ANGLE ON PITCH CYLINDER(RAD) LAMDP1=',F14.7/
&2X,'FACE WIDTH               (MM)       FW1=',F14.7/
&2X,'RADIUS OF PITCH CYLINDER   (MM)     RPT1=',F14.7/
&2X,'RADIUS OF BASE CYLINDER    (MM)     RBT1=',F14.7/
&2X,'RADIUS OF ADDENDUM CYLINDER (MM)    RAT1=',F14.7/
&2X,'RADIUS OF DEDENDUM CYLINDER (MM)    RDT1=',F14.7/
&2X,'TRANSVERSE PRESSURE ANGLE (RAD>     PSIT1=',F14.7/
&2X,'HELIX ANGLE ON BASE CYLINDER (RAD) LAMDB1=',F14.7/)
C...
      RETURN
      END
C...
C ...THE SUBROUTINE  IS FOR DATA OF THEORITICAL GEAR SURFACE
C...
      SUBROUTINE DATAT2
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 UP2,LAMDP2,LAMDB2
      COMMON /DATT2/ TN2,PN2,PSIN2,BETAP2,ADG2,DEG2,LAMDP2,
     & UP2,FW2,RPT2,RBT2,RAT2,RDT2,PSIT2,LAMDB2
      RPT2=TN2/(2.D0*PN2*DCOS(BETAP2))
      PSIT2=DATAN(DTAN(PSIN2)/DCOS(BETAP2))
      RBT2=RPT2*DCOS(PSIT2)
      RAT2=RPT2+ADG2
      RDT2=RPT2-DEG2
      LAMDB2=DATAN(DTAN(LAMDP2)/DCOS(PSIT2))
      BETAB2=DATAN(DTAN(BETAP2)*DCOS(PSIT2))
      DEL2=0.007
C     WRITE(6,110)
      WRITE(55,110)
 110  FORMAT(/2X,'*******************************************',/
     &       2X,'*            DATA OF GEAR 2                *',/
     &       2X,'*******************************************',/)
      WRITE(55,120) TN2,PN2,PSIN2,BETAP2,ADG2,DEG2,LAMDP2,
     &   FW2,RPT2,RBT2,RAT2,RDT2,PSIT2,LAMDB2,DEL2
  120 FORMAT(2X,'GEAR NUMBER OF TEETH                   TN2=',F14.7/
     &2X,'NORMAL DIAMETRAL PITCH   (1/MM)     PN2=',F14.7/
     &2X,'NORMAL PRESSURE ANGLE    (RAD.)     PSIN2=',F14.7/
     &2X,'LEADING ANGLE OF HELIX'              /
     &4X,'ON PITCH CYLINDER  (RAD)         BETAP2=',F14.7/
     &2X,'ADDENDUM            (MM)            ADG2=',F14.7/
     &2X,'DEDENDUM            (MM)            DEG2=',F14.7/
     &2X,'HELIX ANGLE ON PITCH CYLINDER(RAD) LAMDP2=',F14.7/
     &2X,'FACE WIDTH          (MM)            FW2=',F14.7/
     &2X,'RADIUS OF PITCH CYLINDER   (MM)     RPT2=',F14.7/
     &2X,'RADIUS OF BASE CYLINDER    (MM)     RBT2=',F14.7/
     &2X,'RADIUS OF ADDENDUM CYLINDER (MM)    RAT2=',F14.7/
     &2X,'RADIUS OF DEDENDUM CYLINDER (MM)    RDT2=',F14.7/
     &2X,'TRANSVERSE PRESSURE ANGLE (RAD.)    PSIT2=',F14.7/
     &2X,'HELIX ANGLE ON BASE CYLINDER(RAD) LAMDB2=',F14.7/
     &2X,'ELASTIC APPROACH           (MM)     DEL=',F14.7)
C...
      RETURN
      END
C...
```

```
C... ADDITION OF TWO VECTORS
C...
      SUBROUTINE ADDVEC(VA,VB,VC,DD)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 VA(3),VB(3),VC(3)
      DO 101 I=1,3
        VA(I)=VB(I)+DD*VC(I)
  101 CONTINUE
      RETURN
      END
C...
C... DOT PRODUCT OF TWO VECTOR
C...
      SUBROUTINE DOTVEC(AA,VA,VB)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 VA(3),VB(3)
      AA=0.D0
      DO 102 I=1,3
        AA=AA+VA(I)*VB(I)
  102 CONTINUE
      RETURN
      END
C...
C... CROSS PRODUCT OF TWO VECTOR
C...
      SUBROUTINE CROVEC(VA,VB,VC)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 VA(3),VB(3),VC(3)
      VA(1)=VB(2)*VC(3)-VB(3)*VC(2)
      VA(2)=VB(3)*VC(1)-VB(1)*VC(3)
      VA(3)=VB(1)*VC(2)-VB(2)*VC(1)
      RETURN
      END
C...
C... PRODUCT OF MATRIX AND A VECTOR
C...
      SUBROUTINE MAVEC(VA,MC,VB)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 MC(3,3),VA(3),VB(3)
      DO 103 I=1,3
        VA(I)=0.0
        DO 104 J=1,3
          VA(I)=MC(I,J)*VB(J)+VA(I)
  104   CONTINUE
  103 CONTINUE
      RETURN
      END
C...
C...  PRODUCT OF A VECTOR AND A SCALAR
C...
      SUBROUTINE PDSVEC(VA,VB,T)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 VA(3),VB(3)
      DO 105 I=1,3
        VA(I)=T*VB(I)
  105 CONTINUE
      RETURN
      END
C...
C...  STANDARDIZATION OF A VECTOR
```

```
C...
      SUBROUTINE STDVEC(VA,VB)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 VA(3),VB(3)
      CC=0.D0
      DO 106 I=1,3
        CC=CC+VB(I)**2
  106 CONTINUE
      CN=DSQRT(CC)
      DO 107 I=1,3
        VA(I)=VB(I)/CN
  107 CONTINUE
      RETURN
      END
C...
C...  INPUT A VECTOR TO ANOTHER VECTOR
C...
      SUBROUTINE EQVEC(VA,VB)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 VA(3),VB(3)
      DO 108 I=1,3
        VA(I)=VB(I)
  108 CONTINUE
      RETURN
      END
C...
C...  TRIPLE PRODUCT OF THREE VECTORS
C...
      SUBROUTINE TRIVEC(AA,VA,VB,VC)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 VA(3),VB(3),VC(3),V(3)
      CALL CROVEC(V,VB,VC)
      CALL DOTVEC(AA,VA,V)
      RETURN
      END
C...
C...  TRANSFOR MATRIX
C...
      SUBROUTINE TRANSM(AA,BB)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 AA(3,3),BB(3,3)
      AA(1,1)=BB(1,1)
      AA(1,2)=BB(2,1)
      AA(1,3)=BB(3,1)
      AA(2,1)=BB(1,2)
      AA(2,2)=BB(2,2)
      AA(2,3)=BB(3,2)
      AA(3,1)=BB(1,3)
      AA(3,2)=BB(2,3)
      AA(3,3)=BB(3,3)
      RETURN
      END
C...
C...  PRODUCT OF TWO MATRICES
C...
      SUBROUTINE MAPMA(AA,BB,CC)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 AA(3,3),BB(3,3),CC(3,3)
      AA(1,1)=BB(1,1)*CC(1,1)+BB(1,2)*CC(2,1)+BB(1,3)*CC(3,1)
      AA(1,2)=BB(1,1)*CC(1,2)+BB(1,2)*CC(2,2)+BB(1,3)*CC(3,2)
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```
      AA(1,3)=BB(1,1)*CC(1,3)+BB(1,2)*CC(2,3)+BB(1,3)*CC(3,3)
      AA(2,1)=BB(2,1)*CC(1,1)+BB(2,2)*CC(2,1)+BB(2,3)*CC(3,1)
      AA(2,2)=BB(2,1)*CC(1,2)+BB(2,2)*CC(2,2)+BB(2,3)*CC(3,2)
      AA(2,3)=BB(2,1)*CC(1,3)+BB(2,2)*CC(2,3)+BB(2,3)*CC(3,3)
      AA(3,1)=BB(3,1)*CC(1,1)+BB(3,2)*CC(2,1)+BB(3,3)*CC(3,1)
      AA(3,2)=BB(3,1)*CC(1,2)+BB(3,2)*CC(2,2)+BB(3,3)*CC(3,2)
      AA(3,3)=BB(3,1)*CC(1,3)+BB(3,2)*CC(2,3)+BB(3,3)*CC(3,3)
      RETURN
      END
C..
C...  THE SUBROUTINE IS FOR MISALIGMENT
      SUBROUTINE MIAL(K,DGAMM,LFH)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 LFH(3,3)
      S3=DSIN(DGAMM)
      C3=DCOS(DGAMM)
C ** FOR CROSSING ANGLE MISALIGNMENT
      IF(K.EQ.1) THEN
        LFH(1,1)=-1.D0
        LFH(1,2)=0.D0
        LFH(1,3)=0.D0
        LFH(2,1)=0.D0
        LFH(2,2)=-C3
        LFH(2,3)=-S3
        LFH(3,1)=0.D0
        LFH(3,2)=-S3
        LFH(3,3)=C3
      ENDIF
C ** FOR INTERSECTING ANGLE MISALIGNMENT
      IF(K.EQ.2) THEN
        LFH(1,1)=-C3
        LFH(1,2)=0.D0
        LFH(1,3)=S3
        LFH(2,1)=0.D0
        LFH(2,2)=-1.D0
        LFH(2,3)=0.D0
        LFH(3,1)=S3
        LFH(3,2)=0.D0
        LFH(3,3)=C3
      ENDIF
C
      RETURN
      END
C
C .... COMPUTE THE PRINCIPAL CURVATURES OF MODIFIED cutter SURFACE
C
      SUBROUTINE CURVT(KK)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 EFN(3),EHN(3),W1VT2(3),WV12(3),W2VT1(3),KF,KH,KS,KQ
      REAL*8 KM2,KT2,KFF,KHH
      COMMON/A200/ W1(3),W2(3),W12(3),VT1(3),VT2(3),V12(3)
      COMMON /A210/ EX(3),EF(3),EH(3)
      COMMON /A220/ KF,KH
      COMMON /A230/ ET(3),EM(3)
      COMMON /A300/ ES(3),EQ(3)
      COMMON /A310/ KFF,KHH
      COMMON /A380/ KS,KQ
      EFN(1)= EX(2)*EF(3)-EX(3)*EF(2)
      EFN(2)=-(EX(1)*EF(3)-EX(3)*EF(1))
      EFN(3)= EX(1)*EF(2)-EX(2)*EF(1)
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      EHN(1)= EX(2)*EH(3)-EX(3)*EH(2)
      EHN(2)=-(EX(1)*EH(3)-EX(3)*EH(1))
      EHN(3)= EX(1)*EH(2)-EX(2)*EH(1)
      W1VT2(1)= W1(2)*VT2(3)-W1(3)*VT2(2)
      W1VT2(2)=-(W1(1)*VT2(3)-W1(3)*VT2(1))
      W1VT2(3)= W1(1)*VT2(2)-W1(2)*VT2(1)
      W2VT1(1)= W2(2)*VT1(3)-W2(3)*VT1(2)
      W2VT1(2)=-(W2(1)*VT1(3)-W2(3)*VT1(1))
      W2VT1(3)= W2(1)*VT1(2)-W2(2)*VT1(1)
      WV12(1)= W12(2)*V12(3)-W12(3)*V12(2)
      WV12(2)=-(W12(1)*V12(3)-W12(3)*V12(1))
      WV12(3)= W12(1)*V12(2)-W12(2)*V12(1)
      V12F=0.0
      V12H=0.0
      WNEF=0.0
      WNEH=0.0
      VWN= 0.0
      W1TN=0.0
      W2TN=0.0
      DO 1 I=1,3
        V12F= V12(I)*EF(I)+V12F
        V12H= V12(I)*EH(I)+V12H
        WNEF= W12(I)*EFN(I)+WNEF
        WNEH= W12(I)*EHN(I)+WNEH
        VWN = EX(I)*WV12(I)+VWN
        W1TN= EX(I)*W1VT2(I)+W1TN
1     W2TN= EX(I)*W2VT1(I)+W2TN
C... COMPUTE THE CURVATURE OF THE GENERATED SURFACE
      B13=-KF*V12F-WNEF
      B23=-KH*V12H-WNEH
      B33=-KF*V12F**2-KH*V12H**2+VWN-W1TN+W2TN
      B11=B13**2/B33
      B12=B13*B23/B33
      B22=B23**2/B33
      T1=2.0D00*B13*B23
      T2=B23**2-B13**2-(KF-KH)*B33
      SIG1F=0.5D00*DATAN(T1/T2)
C....PRINCIPAL CURVATURES OF THE GENERATED SURFACE
      IF(DABS(SIG1F).LE.0.1D-5) THEN
        KQ=0.5D0*((KF+KH)+(B13**2+B23**2)/B33
     &      +(B23**2-B13**2-(KF-KH)*B33)/(B33*DCOS(SIG1F)))
        KS=KQ-(B23**2-B13**2-(KF-KH)*B33)/(B33*DCOS(SIG1F))
      ELSE
        KQ=0.50D00*(KF+KH)+0.5D00*(B13**2+B23**2)/B33
     &      +B13*B23/(B33*DSIN(2.0D00*(+SIG1F)))
        KS= KQ-2.0D00*B13*B23/(B33*DSIN(2.0D00*(+SIG1F)))
      ENDIF
      SIGSF=-SIG1F
C....PRINCIPAL DIRECTIONS OF THE GENERATED SURFACE
      DO 2 I=1,3
        EQ(I)= DCOS(SIGSF)*EH(I)-DSIN(SIGSF)*EF(I)
2     ES(I)= DSIN(SIGSF)*EH(I)+DCOS(SIGSF)*EF(I)
      IF (KK .LT. 2) GO TO 100
      Q3=ES(1)*ET(1)+ES(2)*ET(2)+ES(3)*ET(3)
      Q4=EQ(1)*ET(1)+EQ(2)*ET(2)+EQ(3)*ET(3)
      IF(Q3.EQ.Q4) THEN
        Q34=DATAN(Q4)
      ELSE
        Q34=DATAN2(Q4,Q3)
      ENDIF
```

```
      KT2=KS*DCOS(Q34)**2+KQ*DSIN(Q34)**2
      KM2=KS*DSIN(Q34)**2+KQ*DCOS(Q34)**2
      GO TO 120
100   KFF=KS
      KHH=KQ
120   RETURN
      END
C
C .... COMPUTE THE DIMENSIONS AND DIRECTIONS OF THE CONTACT ELLIPSE
C
      SUBROUTINE ELLIP
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 KSIG1,KSIG2,KFF,KHH,KS,KQ
      COMMON /A300/ ES(3),EQ(3)
      COMMON /A380/ KS,KQ
      COMMON /A310/ KFF,KHH
      COMMON /A340/ EFF(3),EHH(3)
      COMMON /A360/ A,B,SI(3),FI(3)
      DEL=0.007D0
      PI=DACOS(-1.D0)
      S1=ES(1)*EFF(1)+ES(2)*EFF(2)+ES(3)*EFF(3)
      S2=EQ(1)*EFF(1)+EQ(2)*EFF(2)+EQ(3)*EFF(3)
      SIGSF=DATAN2(S2,S1)
C....COMPUTE THE DIMENSIONS OF THE CONTACT ELLIPSE (A & B)
      KSIG1=KFF+KHH
      KSIG2=KS+KQ
      G1=KFF-KHH
      G2=KS-KQ
      A=(KSIG1-KSIG2-(G1**2-2.D0*G1*G2*DCOS(2.D0*SIGSF)+G2**2)**0.5)
     &  /4.D0
      B=(KSIG1-KSIG2+(G1**2-2.D0*G1*G2*DCOS(2.D0*SIGSF)+G2**2)**0.5)
     &  /4.D0
      A=(DEL/ABS(A))**0.5
      B=(DEL/ABS(B))**0.5
C....COMPUTE THE ANGLE (ALF1) BETWEEN AXES OF ELLIPSE & PRINCIPLE
      S1=G2*DSIN(2.D0*SIGSF)
      S2=G1-G2*DCOS(2.D0*SIGSF)
      ALF1=0.5D0*DATAN2(S1,S2)
      S3=DSQRT(G1**2-2.D0*G1*G2*DCOS(2.D0*SIGSF)+G2**2)
      SS2=S1/S3
      SC2=S2/S3
      ALF1=DATAN(SS2/(1.D0+SC2))
      ALF1=DABS(ALF1)
C.. AXES OF THE CONTACT ELLIPSE
      DO 100 I=1,3
        SI(I)=DSIN(ALF1)*EFF(I)+DCOS(ALF1)*EHH(I)
        FI(I)=DCOS(ALF1)*EFF(I)-DSIN(ALF1)*EHH(I)
  100 CONTINUE
      RETURN
      END
```

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (*Leave blank*)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	January 1995	Final Contractor Report

4. TITLE AND SUBTITLE

Generation and Computerized Simulation of Meshing and Contact of Modified Involute Helical Gears

5. FUNDING NUMBERS

WU–505–62–36
1L162211A47A

6. AUTHOR(S)

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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

University of Illinois at Chicago
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8. PERFORMING ORGANIZATION REPORT NUMBER

E–9410

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Vehicle Propulsion Directorate
U.S. Army Research Laboratory
Cleveland, Ohio 44135–3191
and
NASA Lewis Research Center
Cleveland, Ohio 44135–3191

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

NASA CR–4644
ARL–CR–221

11. SUPPLEMENTARY NOTES

Project manager, Robert F. Handschuh, Vehicle Propulsion Directorate, U.S. Army Research Laboratory, NASA Lewis Research Center, organization code 0300, (216) 433–3969.

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Unclassified - Unlimited
Subject Category 37

This publication is available from the NASA Center for Aerospace Information, (301) 621–0390.

12b. DISTRIBUTION CODE

13. ABSTRACT ***(Maximum 200 words)***

The design and generation of modified involute helical gears that have a localized and stable bearing contact, and reduced noise and vibration characteristics are described. The localization of the bearing contact is achieved by the mismatch of the two generating surfaces that are used for generation of the pinion and the gear. The reduction of noise and vibration will be achieved by application of a parabolic function of transmission errors that is able to absorb the almost linear function of transmission errors caused by gear misalignment. The meshing and contact of misaligned gear drives can be analyzed by application of computer programs that have been developed. The computations confirmed the effectiveness of the proposed modification of the gear geometry. A numerical example that illustrates the developed theory is provided.

14. SUBJECT TERMS

Gears; Gearteeth; Transmissions

15. NUMBER OF PAGES

76

16. PRICE CODE

A05

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified		